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Journal of the
HYDRAULICS DIVISION
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CALIBRATION OF A SUBMERGED BROAD CRESTED WEIR

Richard A. Smith,[1] M. ASCE

SYNOPSIS

This work is different from previous publications on the problem in that it treats the calibration, or correlation of variables, even though the weir is completely submerged and tailwater phenomena control the discharge. Previous writings treat the calibration for the condition of free over-fall, but for the submerged condition, the discharge coefficients were shown to vary over a wide range in unpredictable fashion. This paper shows that for free overfall the discharge is a function of the head on the weir, but for the submerged condition the discharge is a function of the degree of submergence. After calibration, the computed discharge, when compared with the observed discharge, showed a probable deviation of approximately 2 percent.

INTRODUCTION

Studies of floods in Louisiana entail the computation of discharge over inundated levees or roadbeds. Available formulas appeared inadequate for computing the discharge over the levee or roadbed when considered as a broad crested weir. With available weir formulas, their accuracy was vitally dependent on empirical coefficients that were little understood in many instances. In application, a value was somewhat arbitrarily assumed without knowing whether the chosen value was even approximately correct.

Statement of the Problem

At the request of the State of Louisiana, Department of Public Works, hydraulic model studies were instituted. The purpose was to calibrate the inundated roadbed, acting as a weir, for two conditions of flow: first, when

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the weir controlled the discharge; and second, when the tail-water was high, so that control of the discharge had passed to conditions downstream from the weir. It was assumed that in prototype studies, records of floods would yield elevations of the weir crest, head-water, and tail-water surface as revealed by high-water marks.

Objectives in Analysis

In the analysis of observed data, the specific objectives were threefold:

a) Determine a criterion by which it could be ascertained when control of the regimen passed from the submerged weir to tail-water conditions downstream.

b) Calibrate the head-discharge relationship for the condition when control of the regimen was at the weir.

c) Correlate the variables, head-water elevation, tail-water elevation, depth of flow over the weir crest, and rate of discharge, for the condition when control of the regimen was downstream from the weir.

Description of Models

Previous observation had shown that in submerged broad-crested weirs, trapezoidal in cross section, the weir coefficient varies as a function of the slope of the upstream and downstream faces. Therefore, model studies were conducted for two models having different slopes. One had slopes of 1 on 1; the other had slopes of 1 vertical on 2 horizontal, with the upstream slope being the same as the downstream slope in either case. Other features, common to both models, were as follows: Length of weir normal to the direction of flow, 4.95 feet; width of weir crest parallel to the direction of flow, 0.5 feet; height of weir crest above the bottom of the channel, 0.425 feet. A roughness was established by sprinkling the models with masons sand shortly after they had been painted. A diagram of the test apparatus is shown in Figure 1.

Model Operating Procedure

Head-water and tail-water elevations were controlled by a supply valve and tail gate, respectively. Head-water and tail-water elevations were observed by means of piezometric connections, 1/2 inch diameter, to stilling wells equipped with hook gauges. The depth of flow over the weir crest was observed by means of a point gauge located slightly downstream from the centerline. This point gauge was read when the fluctuating water surface was above the point and below the point for approximately equal intervals of time. (Piezometric connections were not deemed feasible at the weir crest because of the relatively high velocities encountered which could entail erroneous readings due to stagnation pressures, eddies, etc.) Discharge was observed by means of a weighing tank and electric stop-clock, which read directly to one-tenth of a second. The model was operated throughout the range of discharge capacity available, up to 1.0 cfs on the weir five feet long, or 0.2 cfs per foot of weir length. At each rate of discharge, the tailwater elevation was varied through a full cycle of progressively higher and progressively lower elevations in successive runs.

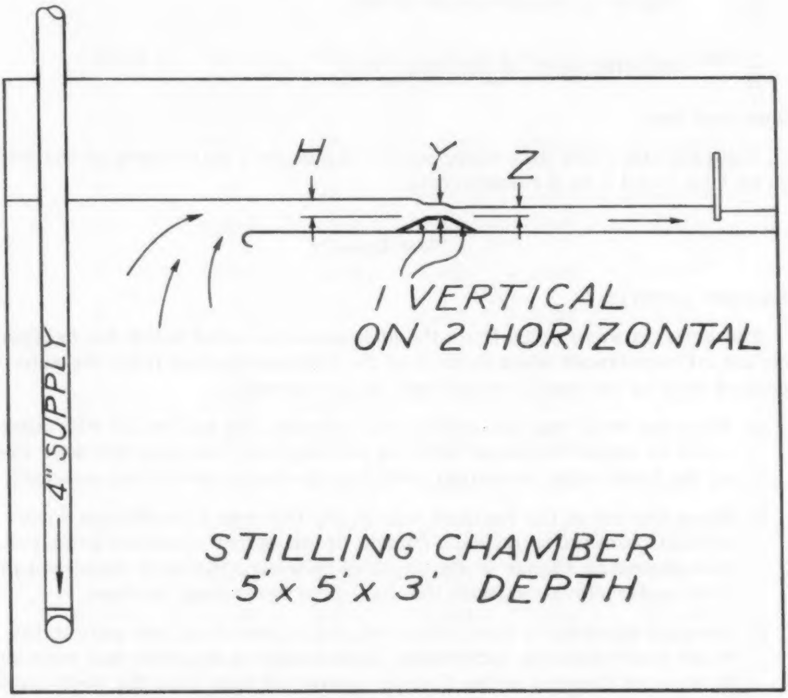

FIG. 1 TEST APPARATUS - SHOWN IN CROSS SECTION

Symbols and Nomenclature

Q	Total discharge rate
q	Discharge per foot length of weir
C	Coefficient of discharge
L	Length of weir, normal to direction of flow
H	Total head, above the weir crest
Y	Depth of flow over the weir crest
z	Tail-water elevation above the weir crest
g	Gravity constant
V	Mean velocity
F	Froude number, $\frac{V}{\sqrt{gy}}$

$\frac{z}{H}$ Degree of submergence of weir

$\frac{y}{H}$ Relative depth at the weir crest

Observed Data

Tables 1 and 2 list data observed for the models with slopes of the weir faces 1 on 1 and 1 on 2 respectively.

Test Results

Regimen Criteria

The criteria were noted from the observations listed below for determining the circumstances when control of the regimen passed from the submerged weir to tail-water conditions, or conversely:

a) When the weir was controlling the regimen, the tail-water elevation could be changed without altering the depth of flow over the weir crest or the head-water elevation, with the discharge remaining constant.

b) When control of the regimen was in the tail-water conditions downstream from the weir, each change in tail-water elevation produced a corresponding change in the depth of flow over the weir crest and in head-water elevation, with the discharge remaining constant.

c) Between successive runs, when control passed from the weir to tail-water conditions, or conversely, this change in regimen was reflected by marked changes in the Froude number of flow over the weir crest.

d) Data from either model indicated that the shift in control from the weir to the tailwater, and conversely, occurred in the region of Froude numbers approximating 0.78 for flow over the weir crest. The higher numbers correspond to the control at the weir, while numbers less than 0.78 correspond to control in the tail-water conditions.

Discharge Coefficients, Weir Controlling

From Figure 2, the head-discharge relationships for the models were combined with the weir formula:

$$Q = CLH^{\frac{3}{2}}$$

Writing the resulting equations with the discharge coefficient as the dependent variable, this yielded the following relationships:

$$C = 3.54H^{.055}$$

for the model having 1 on 1 slopes and,

$$C = 3.64H^{.068}$$

for the model having slopes of 1 on 2. A graph of these coefficients is shown in Figure 4.

TABLE 1 OBSERVED DATA FROM DISCHARGE OVER MODEL OF ROADBED HAVING SIDE SLOPES OF 1 ON 1

RUN NO.	TOTAL HEAD H	CREST DEPTH y	TAILWATER ELEVATION z	DISCHARGE PER FOOT q	OBSERVED COEFFICIENT C	FROUDE NUMBER F
1	0.034	0.025	0.000	.0183	2.92	0.815
2	0.033	0.025	0.000	.0180	3.00	0.805
3	0.053	0.039	0.000	.0369	3.02	0.845
4	0.053	0.038	0.000	.0364	2.98	0.867
5	0.053	0.039	0.023	.0358	2.93	0.819
6	0.053	0.040	0.033	.0356	2.92	0.784
7	0.059	0.053	0.050	.0354	2.46 *	0.465
8	0.123	0.123	0.121	.0351	0.815 *	0.454
9	0.192	0.192	0.191	.0351	0.417 *	0.074
10	0.068	0.064	0.061	.0352	(1.99)*	0.384
11	0.052	0.038	0.000	.0351	2.95	0.836
12	0.074	0.049	0.000	.0610	3.04	0.992
13	0.074	0.050	0.000	.0609	3.03	0.962
14	0.072	0.050	0.009	.0610	3.16	0.962
15	0.075	0.063	0.057	.0608	2.96*	0.670
16	0.104	0.101	0.096	.0611	1.82 *	0.336
17	0.073	0.052	0.039	.0610	3.10	0.908
18	0.072	0.050	0.000	.0609	3.15	0.961
19	0.089	0.065	0.000	.0840	3.17	0.894
20	0.089	0.065	0.000	.0841	3.17	0.896
21	0.089	0.065	0.000	.0841	3.17	0.896
22	0.091	0.068	0.061	.0838	3.04	0.834

TABLE 1 (CONTINUED)

RUN NO.	TOTAL HEAD H	CREST DEPTH y	TAILWATER ELEVATION z	DISCHARGE PER FOOT q	OBSERVED COEFFICIENT C	FROUDE NUMBER F
23	0.096	0.084	0.077	.0839	2.82*	0.608
24	0.090	0.067	0.052	.0842	3.12	0.856
25	0.089	0.066	0.044	.0839	3.16	0.872
26	0.100	0.074	0.000	.1006	3.18	0.882
27	0.100	0.074	0.000	.1004	3.18	0.879
28	0.100	0.074	0.000	.1006	3.18	0.882
29	0.100	0.074	0.000	.1005	3.18	0.881
30	0.109	0.096	0.087	.1002	2.79*	0.594
31	0.130	0.125	0.118	.0976	2.08*	0.390
32	0.129	0.125	0.118	.1003	2.17*	0.400
33	0.104	0.082	0.078	.1006	3.00*	0.756
34	0.103	0.080	0.070	.1011	3.05*	0.789
35	0.101	0.079	0.062	.1011	3.15	0.803
36	0.100	0.078	0.051	.1009	3.19	0.816
37	0.100	0.076	0.025	.1009	3.19	0.850
38	0.100	0.074	0.010	.1011	3.20	0.886
39	0.100	0.073	0.000	.1008	3.19	0.901
40	0.126	0.084	0.000	.1411	3.16	1.022
41	0.126	0.084	0.000	.1400	3.13	1.015
42	0.137	0.118	0.008	.1411	2.78*	0.609
43	0.161	0.154	0.148	.1411	2.19*	0.412
44	0.237	0.236	0.233	.1404	(1.22)*	0.683
45	0.135	0.112	0.100	.1407	2.84*	0.662

TABLE 1 (CONTINUED)

RUN NO.	TOTAL HEAD H	CREST DEPTH y	TAILWATER ELEVATION z	DISCHARGE PER FOOT q	OBSERVED COEFFICIENT C	FROUDE NUMBER F
46	0.129	0.107	0.089	.1417	3.06*	0.714
47	0.126	0.088	0.054	.1405	3.14	0.950
48	0.126	0.084	0.019	.1409	3.15	1.021
49	0.126	0.084	0.003	.1410	3.15	1.022
50	0.126	0.084	0.000	.1410	3.15	1.022
51	0.126	0.084	0.000	.1426	3.19	1.033
52	0.148	0.098	0.000	.1832	3.21	1.053
53	0.149	0.097	0.000	.1831	3.19	1.070

( ) Observed coefficient is markedly different from computed mean, but the observation is in the range of small heads where large percent errors may be anticipated.

* The weir is drowned out, and the head-discharge relationship is controlled by tail-water phenomena.

Discharge Coefficients, Tail-Water Controlling

In Figure 3 is shown graphs of the degree of submergence and relative depth at the weir crest as related to the Froude number. The equations of the curves and the graphs were derived by the method of least squares. In order to avoid undue confusion of the graph, the individual data points are omitted.

For Model with Slopes of 1 on 1

For the model having slopes on the weir faces of 1 on 1, the degree of submergence is related to the Froude number by the equation, obtained by least squares analysis:

$$\left(\frac{Z}{H}\right)^{2.2} + \frac{V^2}{gy} = .980 \qquad (1)$$

TABLE 2 OBSERVED DATA FROM DISCHARGE OVER MODEL OF ROADBED HAVING SIDE SLOPES OF 1 ON 2

RUN NO.	TOTAL HEAD H	CREST DEPTH y	TAILWATER ELEVATION z	DISCHARGE PER FOOT q	OBSERVED COEFFICIENT C	FROUDE NUMBER F
61	0.039	0.024	0.000	.0218	2.83	1.035
62	0.035	0.021	0.000	.0188	2.89	1.091
63	0.053	0.033	0.000	.0349	2.86	1.028
64	0.052	0.033	0.000	.0345	2.90	1.015
65	0.052	0.033	0.032	.0337	2.83	0.991
66	0.058	0.048	0.048	.0332	(2.37)*	0.557
67	0.069	0.045	0.000	.0527	2.91	0.974
68	0.069	0.049	0.046	.0528	2.92	0.859
69	0.072	0.053	0.055	.0529	(2.74)*	0.724
70	0.090	0.058	0.000	.0838	3.10	1.060
71	0.091	0.058	0.028	.0837	3.04	1.057
72	0.101	0.087	0.086	.0837	2.61*	0.575
73	0.101	0.065	0.000	.1014	3.16	1.080
74	0.099	0.065	0.000	.0987	3.16	1.051
75	0.099	0.063	0.005	.0939	3.17	1.103
76	0.099	0.064	0.041	.0987	3.16	1.076
77	0.119	0.108	0.106	.0989	2.41*	0.492
78	0.099	0.065	0.055	.0990	3.17	1.053
79	0.126	0.082	0.015	.1407	3.14	1.059
80	0.126	0.082	0.066	.1412	3.16	1.061
81	0.149	0.136	0.134	.1406	(2.44)*	0.494
82	0.141	0.127	0.123	.1410	2.66 *	0.549

TABLE 2 (CONTINUED)

RUN NO.	TOTAL HEAD H	CREST DEPTH y	TAILWATER ELEVATION z	DISCHARGE PER FOOT q	OBSERVED COEFFICIENT C	FROUDE NUMBER F
83	0.131	0.110	0.104	.1410	2.97*	0.682
84	0.126	0.089	0.089	.1409	3.15	0.936
85	0.126	0.085	0.069	.1411	3.16	1.000
86	0.126	0.082	0.000	.1410	3.16	1.060
87	0.149	0.103	0.093	.1822	3.17	0.973
88	0.149	0.103	0.070	.1781	3.10	0.950
89	0.149	0.101	0.022	.1822	3.17	1.000
90	0.132	0.125	0.121	.1008	2.10*	0.403
91	0.108	0.090	0.091	.1007	2.84 *	0.647
92	0.103	0.086	0.079	.1008	3.05*	0.706
93	0.102	0.069	0.059	.1009	3.10	0.983
94	0.102	0.067	0.046	.1008	3.10	1.025
95	0.101	0.066	0.000	.1009	3.15	1.050
96	0.099	0.084	0.081	.0858	2.75*	0.622
97	0.090	0.070	0.064	.0843	3.12*	0.804
98	0.087	0.058	0.043	.0803	3.12	1.015
99	0.087	0.058	0.000	.0804	3.12	1.016
100	0.100	0.095	0.091	.0631	2.00*	0.380
101	0.077	0.063	0.060	.0622	2.90*	0.694
102	0.073	0.058	0.028	.0620	3.15	0.784
103	0.095	0.094	0.090	.0421	(1.43)*	0.258
104	0.073	0.070	0.065	.0420	2.13 *	0.401
105	0.053	0.036	0.011	.0395	3.29	1.020

TABLE 2 (CONTINUED)

RUN NO.	TOTAL HEAD H	CREST DEPTH y	TAILWATER ELEVATION z	DISCHARGE PER FOOT q	OBSERVED COEFFICIENT C	FROUDE NUMBER F
106	0.053	0.036	0.001	.0395	3.24	1.019

( ) Observed coefficient is markedly different from computed mean, but the observation is in the range of small heads where large percent errors may be anticipated.

* The weir is drowned out, and the head-discharge relationship is controlled by tail-water phenomena.

The relative depth at the weir crest is related to the Froude number by the equation, obtained by least squares analysis:

$$\frac{y}{H} + .334\frac{V^2}{gy} = .990 \; . \tag{2}$$

Combining equations (1) and (2) with the continuity equation,

$$q = Vy$$

the resulting equation is as follows:

$$q = H^{\frac{3}{2}}\sqrt{g\left[.98-\left(\frac{Z}{H}\right)^{2.2}\right]\left[.663+.334\left(\frac{Z}{H}\right)^{2.2}\right]^{3}} \tag{3}$$

By comparing this equation with the weir formula,

$$Q = CLH^{\frac{3}{2}}$$

it is seen that,

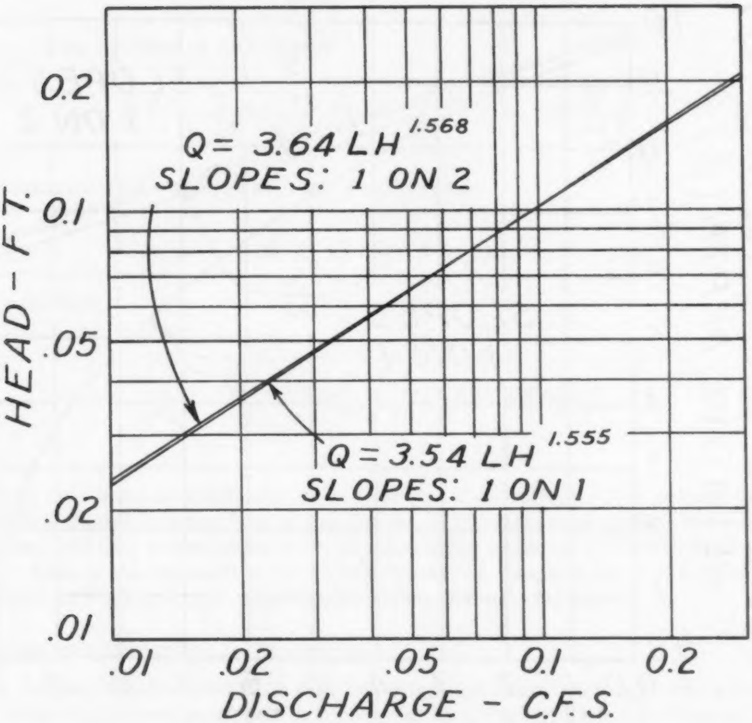

FIG. 2 HEAD-DISCHARGE RELATIONSHIP, CONTROLLED BY MODELS OF ROADBEDS, ACTING AS WEIRS.

$$C = \sqrt{g\left[.98 - \left(\frac{Z}{H}\right)^{2.2}\right]\left[.663 + .334\left(\frac{Z}{H}\right)^{2.2}\right]^{3}} \qquad (4)$$

For Model with Slopes 1 on 2

For the model having slopes on the weir faces of 1 on 2, the degree of submergence is related to the Froude number by the equation, obtained by least squares analysis:

$$\frac{Z}{H} + .334\frac{V^{2}}{gy} = .965 \qquad (5)$$

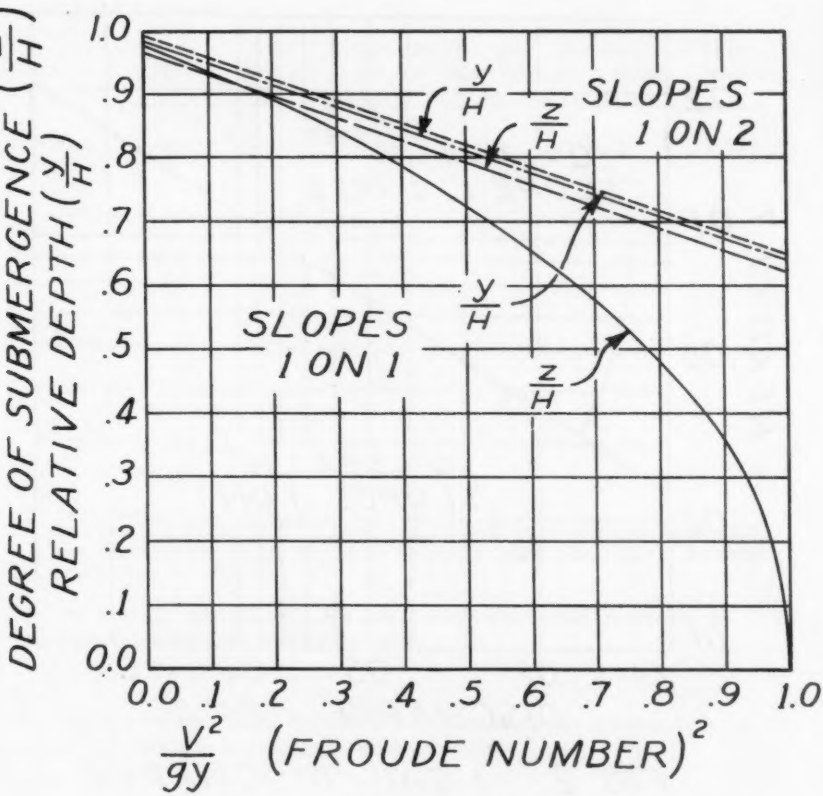

FIG. 3 RELATIONSHIP OF THE FROUDE NUMBER TO DEGREE OF SUBERGENCE AND RELATIVE DEPTH, FOR MODELS OF ROAD-BEDS, ACTING AS SUBMERGED WEIRS.

The relative depth at the weir crest is related to the Froude number by the equation, obtained by least squares analysis:

$$\frac{y}{H} + .334 \frac{V^2}{gy} = .983 \tag{6}$$

When equations (5) and (6) are combined with the continuity equation,

$$q = Vy \ ,$$

the resulting equation is as follows:

$$q = H^{\frac{3}{2}}\sqrt{g\left[2.86 - 2.96\frac{Z}{H}\right]\left[.027 + .991\frac{Z}{H}\right]^3} \ . \tag{7}$$

By comparing this equation with the weir formula:

$$Q = CLH^{\frac{3}{2}} \ ,$$

it is seen that,

$$C = \sqrt{g\left[2.86 - 2.96\frac{Z}{H}\right]\left[.027 + .991\frac{Z}{H}\right]^3} \ . \tag{8}$$

Thus it is seen in equations (4) and (8) that the coefficients of discharge for either model is a function of the degree of submergence (z/H). It should be noted that this correlation is for the condition of the weir drowned out and control of the regimen is in the tail-water. A graph of these coefficients is shown in Figure 4, representing equations (4) and (8), above.

Observations Discarded as Erroneous

In comparing observed data with the computed mean values, it was noted that large deviations occurred in 2 runs out of a total of 53 for one model and in 4 runs out of a total of 46 for the other model. In each instance of large deviation, the total head from head-water to tail-water was 0.018 feet or less. Figure 5 shows that for smaller heads, an error in observed head will produce a larger percent error in computed discharge. Therefore, in arriving at the probable deviation, the larger deviations were omitted from the analysis. Those runs omitted are indicated in Tables 1 and 2, by parentheses enclosing the numerical value of the coefficients in Figure 4.

Errors in Discharge Relative to Errors in Head

The following argument will show the relative deviation in computed discharge caused by a corresponding deviation in observed head. The weir formula,

$$Q = CLH^{\frac{3}{2}} \ ,$$

may be expressed logarithmically,

$$\ln Q = \ln C + \ln L + \frac{3}{2}\ln H \ .$$

If C and L are treated as constants then the derivative of this equation will yield,

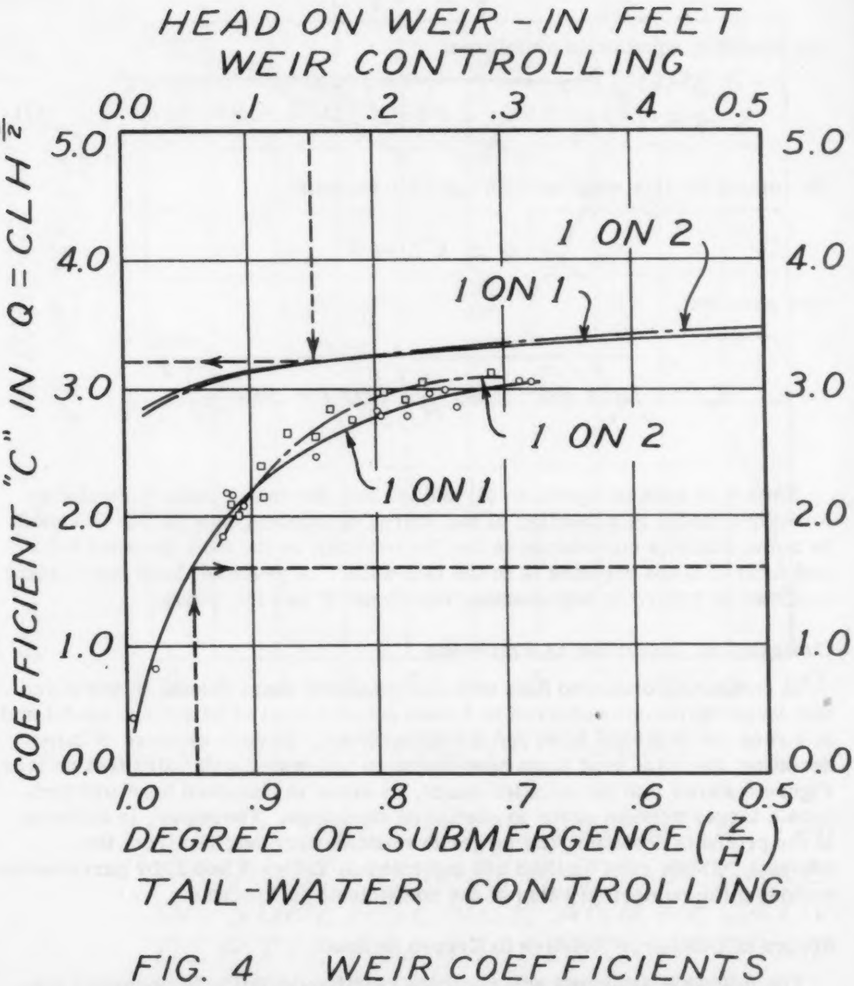

FIG. 4 WEIR COEFFICIENTS FOR MODELS OF ROADBEDS WITH CONTROL EITHER AT THE WEIR, OR IN TAIL-WATER PHENOMENA.

$$\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}$$

For small, finite increments, this may be expressed as follows:

$$\frac{\Delta Q}{Q} = \frac{3}{2} \cdot \frac{\Delta H}{H}$$

The percent deviation in computed discharge is 1.5 times the percent deviation in observed head. This relationship is shown graphically in Figure 5.

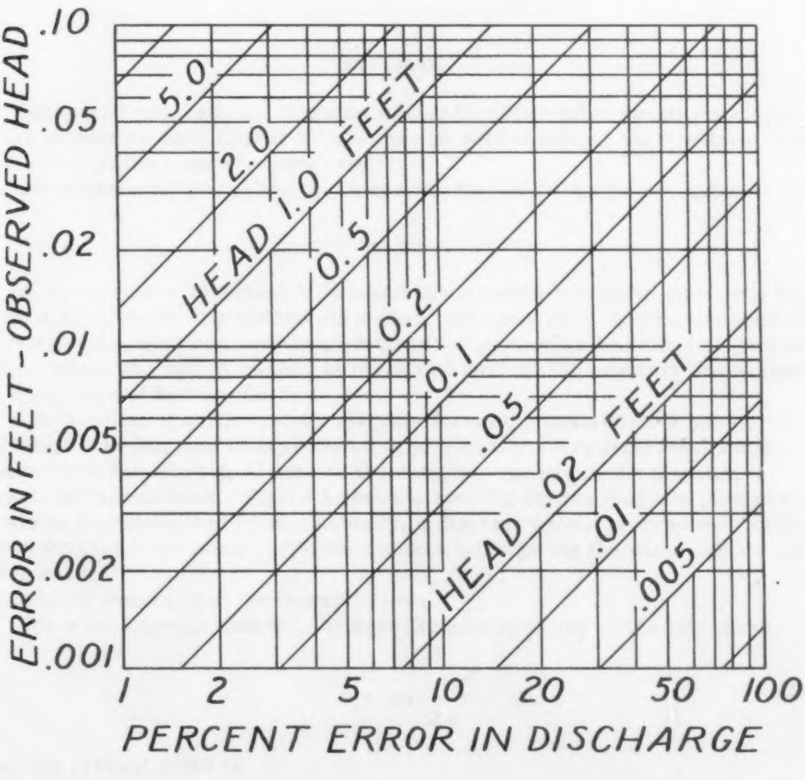

FIG. 5 RELATIVE ERRORS IN OBSERVED HEAD AND COMPUTED DISCHARGE, AS A FUNCTION OF THE HEAD.

Deviation of Observed Data From Computed Mean

The laws or probability were applied to compare the observed discharge with mean values computed by the weir formula and using coefficients from Figure 4. This comparison indicated that the observed data agreed with the computed mean values with a probable deviation of a plus or minus 1.4 percent for the model with the slopes of 1 on 1. Similarly the probable deviation was a plus or minus 2.1 percent for the model with slopes of 1 on 2.

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TABULAR SOLUTION OF OPEN CHANNEL FLOW EQUATIONS

Henry A. Babcock,[1] A.M. ASCE

ABSTRACT

This paper presents and explains a table by which cubic equations which arise in open channel flow problems may be solved without the tiresome trial and error method usually necessary.

Use of the table is limited to the special case of rectangular channels.

Most engineers will cheerfully admit their inability to solve equations beyond quadratics by any method other than "cut-and-try." Special methods for solving cubic equations have been devised for particular equations of frequent occurrence, and such a method is presented here for an equation that arises in open channel hydraulics.

The familiar specific-energy diagram forms the basis for the solution. Although this diagram is reproduced in practically every fluid mechanics textbook on the market,(1) none of them explain the use of the diagram in practical calculations. Figure 1 shows a specific energy diagram plotted to scale in dimensionless form. Actually, a diagram cannot be read with sufficient precision for many purposes and so a table giving the same data is also presented.

A brief summary of the theory follows.

For a rectangular channel, the specific energy at any cross section is

$$E_s = d + \frac{V^2}{2g} \tag{1}$$

and the critical depth is

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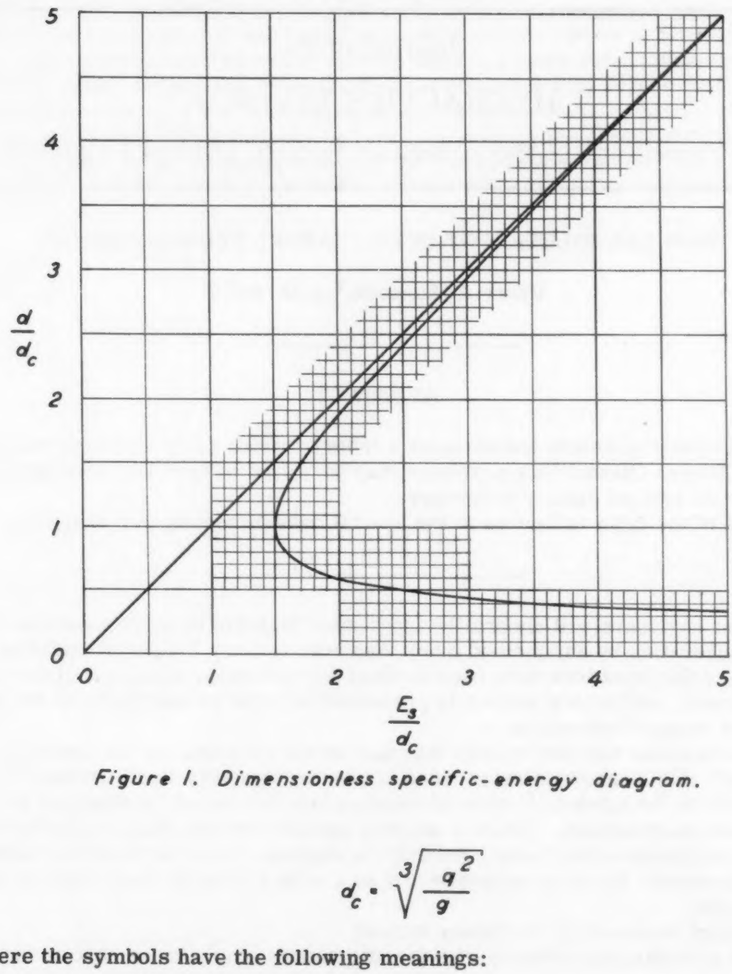

Figure 1. Dimensionless specific-energy diagram.

$$d_c = \sqrt[3]{\frac{q^2}{g}} \tag{2}$$

where the symbols have the following meanings:

E_s = specific energy

d = depth of flow

q = rate of flow, in cfs per foot width

g = acceleration of gravity

d_c = critical depth

Division of Eq. 1 by Eq. 2 gives

$$\frac{E_s}{d_c} = \frac{d}{d_c} + \frac{1}{2}\left(\frac{d_c}{d}\right)^2 \tag{3}$$

Values of $\frac{E_s}{d_c}$ for values of $\frac{d}{d_c} \leqslant 4.99$ are given in Table 1.

Figure 1 was plotted from the data in Table 1.

Use of the diagram will usually yield results of satisfactory accuracy, however, greater mathematical precision is obtainable from the table.

Problems are solved by writing Bernoulli's Equation in the usual way, and then by appropriate manipulation reducing it to the form of Eq. 3. When this is done, one side or the other of Eq. 3 can be evaluated from given data, and then the other side can be found in Table 1. Finally, multiplication of tabular values by d_c will yield a numerical result.

If it is necessary to include an energy loss between two cross sections, the table or graph may be used by making a slight modification of the theory outlined above. Say that between the upstream section and the downstream section there is a loss equal to $K \frac{V_2^2}{2g}$. (The subscript 2 refers to the downstream section.) Bernoulli's theorem written between the upstream section 1 and section 2 is

$$E_{s-1} = d_2 + \frac{V_2^2}{2g} + K \frac{V_2^2}{2g} \tag{4}$$

For rectangular channels Eq. 4 may be simplified to

$$E_{s-1} = d_2 + (1 + K) \frac{q^2}{2 g d_2^2} \tag{5}$$

Setting the derivitive of (5) with respect to d_2 equal to zero and solving for d_2 yields

$$d_2 = \sqrt[3]{\frac{(1+K)q^2}{g}} \tag{6}$$

This value of d_2 will be designated d_m. The relationship between d_m and d_c is

$$d_m = \sqrt[3]{(1+K)}\, d_c \tag{7}$$

Division of Eq. 5 by Eq. 7 gives

$$\frac{E_{s-1}}{d_m} = \frac{d_2}{d_m} + \frac{1}{2}\left(\frac{d_m}{d_2}\right)^2 \tag{8}$$

This equation is of the same form as Eq. 3 and when either side is known the other side may be found from Table 1 or Figure 1.

Illustrative Problems

The following problems would lead to a cubic equation if the accompanying table were not used. Computations are carried to three decimal places to assure accuracy of the second decimal place.

TABLE I

E_s/d_c AS A FUNCTION OF d/d_c

d/dc	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		5000.	1250.	555.6	312.5	200.1	138.9	102.1	78.21	61.82
0.1	50.10	41.43	34.84	29.71	25.65	22.37	19.69	17.47	15.61	14.04
0.2	12.70	11.55	10.55	9.682	8.920	8.250	7.656	7.129	6.658	6.235
0.3	5.856	5.513	5.203	4.921	4.665	4.432	4.218	4.022	3.843	3.677
0.4	3.525	3.384	3.254	3.134	3.023	2.919	2.823	2.734	2.650	2.572
0.5	2.500	2.432	2.369	2.310	2.255	2.203	2.154	2.109	2.066	2.026
0.6	1.989	1.954	1.921	1.890	1.861	1.833	1.808	1.784	1.761	1.740
0.7	1.720	1.702	1.684	1.668	1.653	1.639	1.626	1.613	1.602	1.591
0.8	1.581	1.572	1.564	1.556	1.549	1.542	1.536	1.531	1.526	1.521
0.9	1.517	1.514	1.511	1.508	1.506	1.504	1.502	1.501	1.501	1.500
1.0	1.500	1.500	1.501	1.501	1.502	1.504	1.505	1.507	1.509	1.511
1.1	1.513	1.516	1.519	1.522	1.525	1.528	1.532	1.535	1.539	1.543
1.2	1.547	1.552	1.556	1.560	1.565	1.570	1.575	1.580	1.585	1.590
1.3	1.596	1.601	1.607	1.613	1.618	1.624	1.630	1.636	1.642	1.649
1.4	1.655	1.662	1.668	1.674	1.681	1.688	1.695	1.701	1.708	1.715
1.5	1.722	1.729	1.736	1.744	1.751	1.758	1.766	1.773	1.780	1.788
1.6	1.795	1.803	1.810	1.818	1.826	1.834	1.841	1.849	1.857	1.865
1.7	1.873	1.881	1.889	1.897	1.905	1.913	1.921	1.930	1.938	1.946
1.8	1.954	1.963	1.971	1.979	1.988	1.996	2.004	2.013	2.022	2.030
1.9	2.038	2.047	2.056	2.064	2.073	2.082	2.090	2.099	2.108	2.116
2.0	2.125	2.134	2.142	2.151	2.160	2.169	2.178	2.187	2.196	2.204
2.1	2.213	2.222	2.231	2.241	2.249	2.258	2.267	2.276	2.285	2.294
2.2	2.303	2.312	2.322	2.330	2.340	2.349	2.358	2.367	2.376	2.385
2.3	2.394	2.404	2.413	2.422	2.431	2.440	2.450	2.459	2.468	2.478
2.4	2.487	2.496	2.505	2.515	2.524	2.533	2.543	2.552	2.561	2.571
2.5	2.580	2.589	2.599	2.608	2.618	2.627	2.636	2.646	2.655	2.665
2.6	2.674	2.683	2.693	2.702	2.712	2.721	2.731	2.740	2.750	2.759
2.7	2.769	2.778	2.788	2.797	2.807	2.816	2.826	2.835	2.845	2.854
2.8	2.864	2.873	2.883	2.892	2.902	2.912	2.921	2.931	2.940	2.950
2.9	2.960	2.969	2.979	2.988	2.998	3.008	3.017	3.027	2.036	3.046
3.0	3.056	3.065	3.075	3.084	3.094	3.104	3.113	3.123	3.133	3.142
3.1	3.152	3.162	3.171	3.181	3.191	3.200	3.210	3.220	3.229	3.239
3.2	3.249	3.258	3.268	3.278	3.288	3.297	3.307	3.317	3.326	3.336
3.3	3.346	3.356	3.365	3.375	3.385	3.395	3.404	3.414	3.424	3.434
3.4	3.443	3.453	3.463	3.472	3.482	3.492	3.502	3.512	3.521	3.531
3.5	3.541	3.551	3.560	3.570	3.580	3.590	3.600	3.609	3.619	3.629
3.6	3.639	3.648	3.658	3.668	3.678	3.688	3.697	3.707	3.717	3.727
3.7	3.737	3.746	3.756	3.766	3.776	3.786	3.795	3.805	3.815	3.825
3.8	3.835	3.844	3.854	3.864	3.874	3.884	3.894	3.903	3.913	3.923
3.9	3.933	3.943	3.953	3.962	3.972	3.982	3.992	4.002	4.012	4.021
4.0	4.031	4.041	4.051	4.061	4.071	4.080	4.090	4.100	4.110	4.120
4.1	4.130	4.140	4.150	4.159	4.169	4.179	4.189	4.199	4.209	4.218
4.2	4.228	4.238	4.248	4.258	4.268	4.278	4.288	4.297	4.307	4.317
4.3	4.327	4.337	4.347	4.357	4.366	4.376	4.386	4.396	4.406	4.416
4.4	4.426	4.436	4.446	4.456	4.465	4.475	4.485	4.495	4.505	4.515
4.5	4.525	4.535	4.544	4.554	4.564	4.574	4.584	4.594	4.604	4.614
4.6	4.624	4.634	4.643	4.653	4.663	4.673	4.683	4.693	4.703	4.713
4.7	4.723	4.732	4.742	4.752	4.762	4.772	4.782	4.792	4.802	4.812
4.8	4.822	4.832	4.842	4.851	4.861	4.871	4.881	4.891	4.901	4.911
4.9	4.921	4.931	4.941	4.951	4.960	4.970	4.980	4.990	5.000	5.010

Example 1

At what two depths could a flow of 400 cubic feet per second at a specific head of 7 feet be carried by a rectangular channel 9 feet wide?(2)

From the given data compute the following:

$$d_c = \sqrt[3]{\frac{44.44^2}{g}} = 3.945 \text{ ft.}$$

$$\frac{E_s}{d_c} = \frac{7}{3.945} = 1.774$$

From Table 1, for this value of $\frac{E_s}{d_c}$ the two values of $\frac{d}{d_c}$ are $\frac{d}{d_c} = 1.571$ and $\frac{d}{d_c} = 0.674$. Multiply these values by d_c to obtain

d_t = 6.198 feet (tranquil flow)

d_r = 2.659 feet (rapid flow)

Example 2

A channel 10 feet wide and 6 feet deep carries 300 cubic feet per second. A transition reduces the width to 9 feet with no change in bottom elevation. Find the depth in the narrow section.

From the given data compute the following:

In the 10 ft. wide section

$$d_{c-1} = \sqrt[3]{\frac{30^2}{g}} = 3.036 \text{ ft.}$$

$$\frac{d_1}{d_{c-1}} = \frac{6}{3.036} = 1.976$$

Enter Table 1 with this value of $\frac{d_1}{d_c}$ and obtain $\frac{E_{s-1}}{d_{c-1}} = 2.104$. Hence $E_{s-1} = 2.104 \times 3.036 = 6.388$ feet.

In the narrower section

$$d_{c-2} = \sqrt[3]{\frac{33.33^2}{g}} = 3.257 \text{ ft.}$$

If losses are neglected $E_{s-2} = E_{s-1} = 6.388$, and $\frac{E_{s-2}}{d_{c-2}} = \frac{6.388}{3.257} = 1.961$.

From Table 1 the corresponding value of $\frac{d_2}{d_c} = 1.808$ from which

$$d_2 = 1.808 \times 3.257 = 5.889 \text{ feet.}$$

Example 3

If a rectangular channel 8 feet wide carries a flow of 250 cubic feet per second at a depth of 5 feet, what change in surface elevation will be produced by a local rise in floor level of 1/2 foot?

$$d_c = \sqrt[3]{\frac{31.25^2}{g}} = 3.118 \text{ ft.}$$

$$\frac{d_1}{d_c} = \frac{5}{3.118} = 1.603$$

Enter Table 1 with this value of $\frac{d_1}{d_c}$ to obtain $\frac{E_1}{d_c} = 1.797$. From Bernoulli's theorem

$$E_{s-1} = E_{s-2} + 0.5$$

Divide both sides of this equation by d_c and rearrange the terms to obtain

$$\frac{E_{s-2}}{d_c} = \frac{E_{s-1}}{d_c} - \frac{0.5}{d_c}$$

All quantities on the right of this equation are known.

$$\frac{E_{s-2}}{d_c} = 1.797 - 0.160 = 1.637$$

Enter Table 1 with this value and obtain $\frac{d_2}{d_c} = 1.371$ from which $d_2 =$ 4.28 ft. Hence the surface will drop 0.22 ft.

Exanple 4

The bottom of a rectangular flume is 10.00 feet below the pool level in a reservoir. If the entrance loss is 0.2 $\frac{V_2}{2_g}$ find the water depth in the flume for a flow of 70 cubic feet per second per foot.

Write Bernoulli's theorem between the pool surface and the channel

$$H = d + \frac{V^2}{2g} + 0.2\frac{V^2}{2g}$$

Substitute $V = \frac{q}{d}$ and combine the last two terms.

$$H = d + 1.2\frac{q^2}{2gd^2}$$

Let $d_m = \sqrt[3]{\frac{1.2q^2}{g}}$ and divide each side by d_m. The result

$$\frac{H}{d_m} = \frac{d}{d_m} + \frac{1}{2}\left(\frac{d_m}{d}\right)^2$$

This is the same form as Equation 3 and Table 1 may be used with d_m replacing d_c. Therefore, compute

$$d_m = \sqrt[3]{1.2\,\frac{70^2}{32.2}} = 5.669 \text{ ft.}$$

$$\frac{H}{d_m} = \frac{10}{5.669} = 1764$$

Enter the table with this value of $\frac{H}{d_m}$ and obtain $\frac{d}{d_m} = 1.558$ or $d = 1.558 \times 5.669 = 8.83$ ft.

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RELATIONSHIPS BETWEEN PIPE RESISTANCE FORMULAS

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SYNOPSIS

The relationship between modern concepts of pipe resistance and the older empirical formulas is clarified, and a simple procedure developed to derive an exponential formula applicable to a known range of flow conditions. The analysis makes apparent the range of flow conditions for which any particular exponential formula is applicable. Limitations of the equivalent pipe concept are discussed. Data on resistance measurements in water mains in service indicate that the head loss varies with the discharge to a power nearly equal to 2.0 rather than 1.85 as commonly assumed.

INTRODUCTION

Methods for predicting the resistance encountered by a fluid flowing through a pipe have long been of concern to mathematicians and engineers. The problem has been recognized as an important one in the design of water supply lines and water distribution systems as well as in process plants of various types. Recently the use of long pipelines for transporting large volumes of fluids has renewed interest in problems of pipe resistance.

Over the years, different approaches have been made to the problem of predicting pipe resistance and a multitude of equations and graphs have been devised for evaluating the resistance. In recent years, the literature on the subject presents semi-rational approaches leading to relations applicable over a wide range of conditions. These approaches have been offered to replace the older empirical relations. Many practicing engineers, however, cling to the old familiar relationships without a clear realization of their limitations. Perhaps this should not be surprising because so little has been written to explain the interrelationship between the various methods of

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evaluating resistance. This paper attempts to clarify this interrelationship showing exactly how the older empirical equations may be regarded as special cases of the more recent general resistance relations. It is hoped that with clarification of this interrelationship more use will be made of the recent information as it is presented in the general resistance diagram. This information may be used in equations of the same exponential form as the common empirical formulas by means of the principles presented herein.

Development of Pipe Resistance Relations

A brief review of some selected major steps in the development of pipe resistance relations will help to clarify the resistance relations to which reference is made. In Table I, some selected major steps in the development of pipe resistance relations are shown giving the approximate date of the development, the name of the investigator commonly associated with it, the resistance relation, and the conditions for its application.

Some of the resistance relations, namely the Chezy, Manning, and Hazen-Williams equations, are customarily in a form convenient for solving for the velocity V. In Table I, these are also written in a form in which the head loss h_f is expressed in terms of the other variables after substituting for the slope S as $\frac{h_f}{L}$. In this form, they are more comparable to the common form of the Darcy-Weisbach equation. The relations listed as previous to 1930-31 in Table I are all based on empirical studies with the flow of water. They can all be expressed in the general form

$$h_f = \frac{CL}{R^z} V^m \tag{1}$$

where C is a coefficient to account for the pipe material, L the length of the pipe, R the hydraulic radius, V the mean velocity, and z and m are exponents. More generally they could all be expressed as

$$h_f = KQ^m \tag{2}$$

where K is a coefficient which depends upon the length, diameter, and material of the pipe and Q is the discharge. Equations of this type are termed exponential equations because the head loss is expressed in terms of the pipe characteristics as embodied in K and the discharge Q raised to some exponent m.

The modern understanding of pipe resistance began about 1930 with the Prandtl-Karman relations which were supported by the experiments of Nikuradse, Colebrook, and White. These modern relations are based on an understanding of the variation of the resistance coefficient f in the Darcy-Weisbach equation. For the steady uniform flow of an incompressible fluid the complete picture is well summarized in a general resistance diagram such as proposed by Rouse and by Moody in slightly different forms. The values of f from the general resistance diagram are substituted in the Darcy-Weisbach equation

TABLE I

Selected Major Steps in the Development of Pipe Resistance Relations

Year	Name	Relation	Conditions for Application
1775	Chezy	$V = C\sqrt{RS}$ $h_f = LV^2/RC^2$	Flow of water in open channels.
1845	Darcy-Weisbach	$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$	All pipe flow.
1859	Poiseuille	$h_f = \frac{32\mu VL}{\gamma D^2}$	Laminar flow in pipes.
1883	Reynolds	$N_R = \frac{VD}{\nu} < 2000$	Criteria for Laminar Flow in pipes.
1889	Manning	$V = \frac{1.49}{n} R^{2/3} S^{1/2}$ $h_f = \frac{n^2 LV^2}{2.21 R^{2/3}}$	Turbulent flow of water in channels and pipes.
1892	Freeman	Exhaustive tests and tabular results.	Flow of water in commercial pipes.
1914	Hazen-Williams	$V = 1.318 C_1 R^{0.63} S^{0.54}$ $h_f = \frac{LV^{1.85}}{1.66 C^{1.85} R^{1.165}}$	Turbulent flow of water in pipes.
1930-31	Prandtl-Karman	$\frac{1}{\sqrt{f}} = 2 \log N_R \sqrt{f} - 0.8$ $\frac{1}{\sqrt{f}} = 2 \log \frac{r_o}{e} + 1.74$	Turbulent flow of any fluid in a "smooth" pipe." Turbulent flow of any fluid in a "rough" pipe.
1932-33	Nikuradse	Systematic test with controlled roughness.	Pipes artifically roughened with sand grains.
1939	Colebrook-White	$\frac{1}{\sqrt{f}} = -2 \log \frac{2.51}{N_R\sqrt{f}} + \frac{e}{3.7D}$	Commercial pipe, turbulent flow, "smooth," "rough," and transition conditions.
1942	Rouse	Plot of $f = \phi\,(N_R,\ N_R\sqrt{f}, \frac{e}{D})$	General resistance diagram for commercial pipe: Laminar and turbulent flow, smooth, rough, and transition conditions.
1944	Moody	Plot of $f = \phi\,(N_R, \frac{e}{D})$	"Moody Diagram" - for commercial pipe: Laminar and turbulent flow, smooth, rough, and transition conditions.

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} \tag{3}$$

to evaluate the flow resistance. This relation may also be expressed as

$$h_f = K_1 Q^2 \tag{4}$$

where K_1 depends on the length, diameter, and roughness of the pipe as well as the Reynolds number of the flow.

Each of the two types of formulas, the exponential type and the one based

on the general resistance diagram, has some advantages as well as disadvantages. With the exponential type, the calculations are convenient; and for this reason it is widely used by many practicing engineers. The calculations are direct, avoiding the need for trial-and-error solutions. The reliability of the calculated resistance is questionable, however, because of the uncertainty in selecting the applicable equation from the very large number of exponential formulas that have been proposed. As late as 1951, Blair[10] refers to the multitude of formulas already proposed and proceeds to suggest four new exponential formulas designed to cover a wide range of pipe materials. Again in 1954[14] these formulas were reexamined and eleven new exponential formulas (divided into four groups) were proposed to represent a wide range of conditions. Because the exponential formulas contain no term involving the fluid viscosity, they are necessarily limited to conditions where viscosity has little effect or to a restricted range of Reynolds numbers.

The modern general resistance diagram, however, accounts in a rational manner for all of the factors which influence pipe resistance. A clear understanding of the mechanism of pipe resistance has come from this approach, and the general resistance diagram clearly shows the effect of the various factors entering into the problem. The improved knowledge incorporated in the general resistance diagram, however, has not received as wide usage among practicing engineers as it deserves. This is probably largely due to its inconvenience as compared with the simpler exponential formulas. With the general resistance diagram in its common form, trial-and-error solutions are required for finding the rate of flow or the size of pipe.

Because of the convenience of the exponential formulas and their widespread usage, it is desirable that the interrelationship between them and the general resistance diagram be clearly understood. In this way it may be possible to benefit from the convenience of the exponential formulas as well as the completeness of the general resistance diagram. The interrelationship between exponential formulas and the general resistance diagram has not previously been presented in complete form.

Interrelationship Between Moody Diagram and Exponential Formulas

For a limited range of Reynolds numbers and a fixed value of $\frac{e}{D}$, the plot of log f versus log N_R may be approximated by a straight line and f may be expressed in terms of the Reynolds number N_R as

$$f = \frac{C_o}{N_R{}^{(2-m)}} \tag{5}$$

where C_o is a constant and (2-m) represents the negative slope of the plot of log f versus log N_R on the Moody diagram. If this is substituted into a Darcy-Weisbach equation and the Reynolds number is expressed in terms of the mean velocity, pipe diameter, and kinematic viscosity of the fluid, there results

$$h_f = \frac{C_o \nu^{(2-m)} L}{2g(\pi/4)^m D^{3+m}} \quad Q^m \tag{6}$$

The head loss may then be expressed as

$$h_f = K_o Q^m \tag{7}$$

where

$$K_o = \frac{C_o \nu^{(2-m)} L}{2g(\pi/4)^m D^{3+m}} \tag{8}$$

In this expression both the coefficient C_o and the exponent m will be functions of the Reynolds number and the relative roughness of the pipe $\frac{e}{D}$. Thus, when the head loss is expressed in the form of equation (7), the coefficient K_o depends upon the pipe characteristics of length, diameter, and relative roughness as well as the Reynolds number and fluid viscosity (see equation (8)).

The exponent m may be conveniently determined from the Moody diagram because (2-m) is the negative slope of a curve on the Moody diagram. Thus lines of constant m may be drawn on the Moody diagram by connecting points of constant slope. This has been done in the modified Moody diagram of Fig. 1.

The modified Moody diagram makes clear why so many exponential formulas have been proposed. It is evident that to completely represent the resistance problem an infinite family of exponential formulas is needed with the exponent m varying continuously from a minimum value of about 1.7 for smooth pipes at a Reynolds number of about 4000 to a maximum value of 2 for a relatively rough pipe at large Reynolds numbers. For a given value of Reynolds number and relative roughness the proper value of the exponent may be read from the modified Moody diagram.

The use of the proper exponent m is important when the head loss is to be determined for different discharges or when a change of head loss is to be related to a change in discharge as in the Hardy Cross network analysis. Differentiation of the equation $h_f = KQ^m$ to yield $\frac{dh_f}{h_f} = m\frac{dQ}{Q}$ shows that the relative change in head loss is m times the relative change in discharge. Any of the common exponential equations such as the Hazen-Williams, Manning, or Darcy-Weisbach (with constant f) may be made to yield the correct head loss at a given discharge by proper choice of C, n, or f. However, the head loss at other discharges as well as the relation between a change in head loss and a change in discharge will be incorrect unless the exponent m of the selected formula corresponds to the combination of relative roughness and Reynolds number for the pipe under consideration. The modified Moody diagram gives an easy check on the proper m value for any combination of relative roughness and Reynolds number. Thus the Hazen-Williams formula with its exponent of 1.85 corresponds to a combination of relative roughness and Reynolds number lying along the line m = 1.85 on the Moody diagram. The Manning equation or the Darcy-Weisbach (with constant f) corresponds to conditions lying on or to the right of the line m = 2.0.

With the aid of the modified Moody diagram it is convenient to obtain an exponential formula for a reasonable range of conditions which will accurately reflect the resistance characteristics as embodied in the Moody diagram. The following steps have been found most convenient:

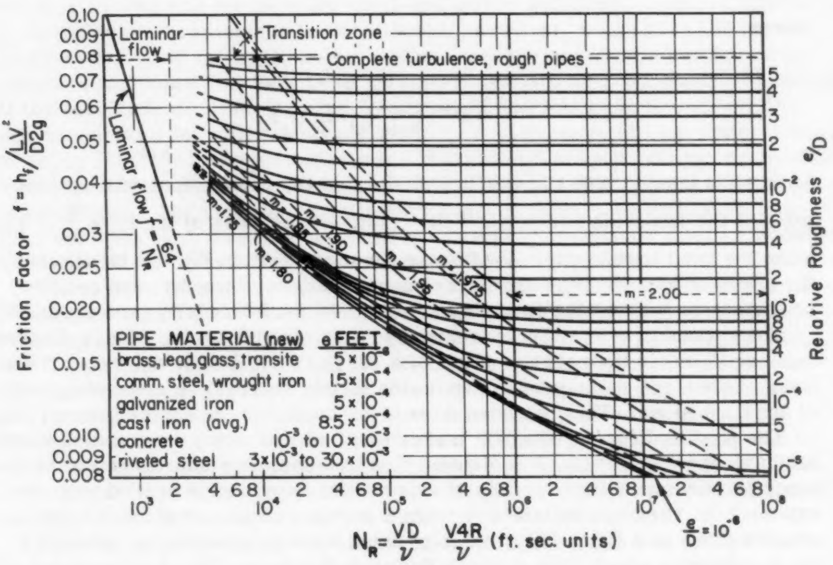


Fig. 1 Modified Moody Diagram

1. Select approximate values of Reynolds number N_R and relative roughness $\frac{e}{D}$.
2. From the modified Moody diagram obtain the exponent m and the friction factor f.
3. For a velocity consistent with the Reynolds number used in Step 1 calculate the head loss in the pipe using the friction factor f in the equation.

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{9}$$

Compute the discharge Q in the pipe.

4. Solve for $K_o = \frac{h_f}{Q^m}$.

This gives values for K_0 and m in the equation $h_f = K_0Q^m$. Because K_0 and m may change with Reynolds number and hence with Q the exponential equation thus found should be limited to a reasonable range of variation in Q. For conditions falling in the transition range between smooth-pipe and rough-pipe flow on the general resistance diagram, the results from the exponential equation agree with those obtained from the resistance diagram within 5 per cent for a twenty-fold variation in Q. The agreement will be closer for conditions approaching those for rough-pipe flow, that is, m approaching 2, as well as for conditions approaching smooth-pipe flow. Examples in the Appendix show in detail the method of obtaining exponential formulas from the modified Moody diagram.

Equivalent Pipe Concept

The equivalent pipe concept is commonly used to analyze the flow for pipes in series or in parallel. This may be done by replacing the individual pipes in the system, for purposes of calculation, with several pipes all of the same diameter but whose lengths are chosen to give loss characteristics equivalent to the original pipes. Two pipes are thus considered equivalent if they have equal head loss at equal rates of flow. This may be expressed in terms of the exponential equations as follows:

1. For pipe No. 1 let $h_f = K_{0_1} Q^{m_1}$.
2. For pipe No. 2 let $h_f = K_{0_2} Q^{m_2}$.

In the normal use of the equivalent pipe concept the exponents m_1 and m_2 are assumed equal when a particular exponential equation is selected for the problem. Then if pipe No. 2 is to be made equivalent to pipe No. 1 the length of pipe No. 2 is so chosen that K_{0_2} is equal to K_{0_1}.

The previous discussion of exponential formulas makes it clear that the exponents m may not be equal for all pipes in a system and if this is true there is no one pipe which can be made equivalent to the various pipes in the system. Equivalence may be achieved for one rate of discharge, but not for a broad range of discharge.

The effect of a difference in the exponent m may be illustrated by an example in which a 12-inch diameter steel pipe is compared with a 6-inch diameter cast iron pipe. A 100-foot length of 6-inch cast iron pipe having a resistance relation $h_f = 1.88Q^{1.95}$ may be compared with 3660 ft. of 12-inch steel pipe having a resistance relation $h_f = 1.99Q^{1.88}$. The length of 12-inch pipe was chosen so that the two pipes would be "equivalent," that is the head loss for the two pipes would be equal, at a discharge of 2.25 cfs. The results of this comparison are shown in Table II. It is apparent that while the pipes are equivalent at a discharge of 2.25 cfs, at a flow of 0.5 cfs the head loss in the steel pipe is 12 per cent greater than that in the 6-inch cast iron pipe, while at a flow of 10 cfs the head loss in the 12-inch steel pipe is 11 per cent less than that in the 6-inch cast iron pipe. This demonstrates that when the exponents m differ significantly the equivalent pipe concept may lead to inaccurate results.

Although there are some situations where the variation in the exponent m is significant, in many applications of resistance formulas for design purposes

TABLE II

The Effect of the Exponent m on the Equivalence of Pipes

Q cfs	3660 ft. of 12-inch Steel Pipe $h_{f_{12}} = 1.99 Q^{1.88}$ ft.	100 ft. of 6-inch Cast Iron Pipe $h_{f_6} = 1.88 Q^{1.95}$ ft.	Ratio $h_{f_{12}}/h_{f_6}$
0.5	0.54	0.49	1.12
1.0	1.99	1.88	1.06
2.0	7.32	7.25	1.01
3.0	15.70	16.00	0.98
4.0	27.00	28.20	0.96
6.0	57.70	62.00	0.93
8.0	99.50	109.00	0.91
10.0	151.00	169.00	0.89

the variation in m will be of little practical significance. If a pipe system includes pipes with a broad range of relative roughness and a broad range of Reynolds numbers, the conventional "equivalent pipe" concept may not be adequate to explain the operation of the system. However, its performance could be understood in terms of the applicable resistance equations as determined from the modified Moody diagram. In many applications of resistance formulas for design purposes, the inaccuracy due to the variation in the exponent m will be of little practical significance because the precision of the basic design conditions does not justify great refinement of the calculations. Also, in many applications it will be found that the exponent m will be practically the same for all pipes in the system.

Exponential Equations and Complex Pipe Systems

In complex systems, pipes may occur in series, in parallel, or in networks. The exponential forms of the resistance relations are especially convenient for solving problems of these types.

Consider n pipes in series, each with the corresponding resistance relations

$$\begin{aligned} h_{f_1} &= K_{o_1} Q^{m_1} \\ h_{f_2} &= K_{o_2} Q^{m_2} \\ h_{f_3} &= K_{o_3} Q^{m_3} \\ h_{f_n} &= K_{o_n} Q^{m_n} \end{aligned} \qquad (10)$$

Clearly, the total head loss for the n pipes in series is

$$\sum_{j=1}^{j=n} h_{f_j} = K_{o_1} Q^{m_1} + K_{o_2} Q^{m_2} + \cdots + K_{o_n} Q^{m_n} \tag{11}$$

If it is possible to choose exponential relations such that the actual resistance characteristics of each pipe can be closely approximated using the same exponent m; that is, if

$$m_1 = m_2 = m_3 \cdots\cdots = m_n = m$$

then the total head loss reduces to the simple form:

$$\Sigma h_f = (K_{o_1} + K_{o_2} + K_{o_3} + \cdots\cdots + K_{o_n}) Q^m$$

or

$$\sum_{j=1}^{j=n} h_{f_j} = \left(\sum_{j=1}^{j=n} K_{o_j}\right) Q^m \tag{12}$$

The quantity $\left(\sum_{j=1}^{j=n} K_{o_j}\right)$ may be thought of as an equivalent resistance factor K'_{o_s} so that the total head loss could be expressed as

$$\sum_{j=1}^{j=n} h_{f_j} = K'_{o_s} Q^m \tag{13}$$

Use of the constant exponent m makes this analogous to the "equivalent pipe" method and subject to the same limitations referred to previously. However, the resistance characteristics of each pipe is characterized by the value of K_o rather than by some fictitious length or diameter. Equation (10) permits the direct solution for either the total head loss or the discharge.

If n pipes are arranged in parallel so that the head loss is the same for each pipe then the total discharge through the system is the sum of the discharges through each individual pipe or

$$\text{total } Q_T = \sum_{j=1}^{j=n} Q_j = Q_1 + Q_2 + \cdots Q_n \tag{14}$$

But since for each pipe $h_f = K_o Q^m$ or $Q = \left(\frac{h_f}{K_o}\right)^{1/m}$ the total discharge Q_T may be expressed as

$$Q_T = \left(\frac{h_{f_1}}{K_{o_1}}\right)^{1/m_1} + \left(\frac{h_{f_2}}{K_{o_2}}\right)^{1/m_2} + \cdots + \left(\frac{h_{f_n}}{K_{o_n}}\right)^{1/m_n} \tag{15}$$

in which $h_{f_1} = h_{f_2} = \ldots h_{f_n}$. Again if it is possible to choose an exponential resistance relation so that m is the same for each pipe, a simplification results giving

$$Q_T = h_{f_1}^{\;1/m} \left[\frac{1}{K_{o_1}^{\;1/m}} + \frac{1}{K_{o_2}^{\;1/m}} + \cdots + \frac{1}{K_{o_n}^{\;1/m}} \right] \tag{16}$$

or

$$h_f = \frac{1}{\left[\left(\frac{1}{K_{o_1}}\right)^{1/m} + \left(\frac{1}{K_{o_2}}\right)^{1/m} + \cdots + \left(\frac{1}{K_{o_n}}\right)^{1/m} \right]^m} Q_T^m \tag{17}$$

Thus an equivalent resistance factor K'_{o_p} may then be defined for the system of parallel pipes as

$$K'_{o_p} = \frac{1}{\left[\left(\frac{1}{K_{o_1}}\right)^{1/m} + \left(\frac{1}{K_{o_2}}\right)^{1/m} + \cdots + \left(\frac{1}{K_{o_n}}\right)^{1/m} \right]^m} = \frac{1}{\left[\sum_{j=1}^{j=n} \left(\frac{1}{K_o}\right)^{1/m} \right]^m} \tag{18}$$

The head loss for the system of parallel pipes may then be expressed as

$$h_f = K'_{o_p} \, Q_T^{\;m} \tag{19}$$

If the flow through any one pipe in the system is desired it may be found by equating the head loss in the particular pipe to the head loss through the parallel system

$$h_f = K'_{o_p} Q_T^m = K_{o_n} Q_n^m$$

or

$$Q_n = Q_T \left(\frac{K'_{o_p}}{K_n} \right)^{1/m} \tag{20}$$

Many complex pipe systems may, for the purpose of calculations, be considered as combinations of parallel units and series units so that the entire system can be analyzed by repeated application of the relationships for parallel pipes and series pipes. Thus if the K_o values are found for each pipe in an exponential relationship with a constant exponent, equations (10), (11), (16), and (17) will permit a complete solution in terms of the K_o values for the individual pipes.

A complex pipe system which includes some pipes which are common to adjacent loops can not be considered as a combination of parallel units and

series units and must be analyzed as a network. Although the methods for analysis of networks will not be considered here, it is pertinent to note that exponential resistance relations will be most convenient for use in this type of analysis.

Values of the Exponent m for Water Mains

Although the Hazen-Williams equation which uses an exponent m of 1.85 is commonly used for the design of water mains in city distribution systems, test results appear to indicate that this is more representative of new pipe than of pipe that has been in service some time. It appears that the increased surface roughness on the interior of the pipe not only increases the resistance (giving a lower Hazen-Williams C value) but also requires a change in the exponent m to a value more nearly equal to 2.0.

Lamont(14) presents resistance data selected from more than 200 records of resistance measurements. In Fig. 2, the results from ninety-seven experiments on new pipes of all types are plotted on the modified Moody diagram indicating that many of the results lie in the transition range between smooth and rough conditions with the exponent m varying from 1.80 to 1.90.

Fig. 3 is a similar plot for the approximately 80 records, Lamont reported for "old and slimy" pipes. These results clearly indicate that a larger value of the exponent m is required for these pipes than for new pipes. For most of these records an exponent of nearly 2.0 is indicated.

Recent tests on water mains in Austin, Texas, when plotted on the modified Moody diagram have also indicated that values of the exponent m are nearly 2.0. These results are summarized in Table III.

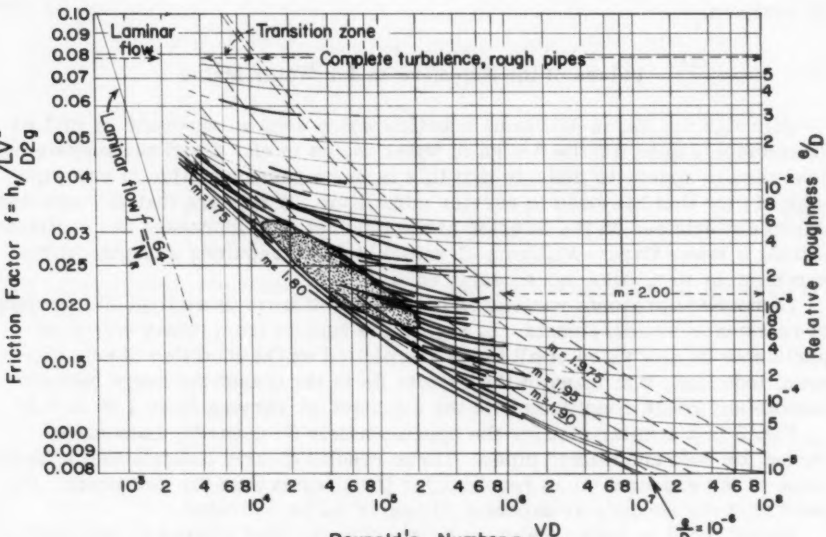
The exponent of nearly 2.0 obtained from resistance tests on old pipes indicates that for pipes that have been in service for some years the resistance characteristics could better be represented by an equation of the Manning type or the Darcy-Weisbach equation with a constant f than by the commonly used Hazen-Williams equation.

CONCLUSIONS

1. The modified Moody diagram with lines of constant exponent m is a convenient aid in determining the exponential type resistance formula applicable to a particular pipe for a limited range of Reynolds numbers.

2. An infinite and continuous family of exponential formulas with the exponent ranging from a minimum of about 1.70 to a maximum of 2.0 is required to correctly represent all pipe flow conditions. This explains why such a multitude of exponential formulas have been presented in the literature on pipe resistance.

3. The commonly used exponential formulas and the equivalent pipe concept as based on these formulas are sufficiently accurate for many practical applications. With some combinations of pipe sizes, roughnesses, and flow rates, resistances may occur which cannot be explained by the common exponential formulas but which can be understood in terms of the concepts here presented.



Note:
In dotted area the lines were too thick to be distinguishable

Fig. 2

Resistance Records on New Pipes

4. Exponential formulas as developed from the modified Moody diagram for a particular system of pipes may be used conveniently in the analysis of series or parallel pipes or a combination of these.

5. Results of resistance tests on water mains as reported herein indicate that for new pipes an exponent m from 1.80 to 1.90 is common which is in fair agreement with the Hazen-Williams formula, however, for old pipes an exponent of nearly 2.0 is common which is in closer agreement to the Darcy-Weisbach or Manning equations.

List of Symbols

A	Cross sectional area of flow passage - sq. ft.
C	Coefficient in the Chezy equation

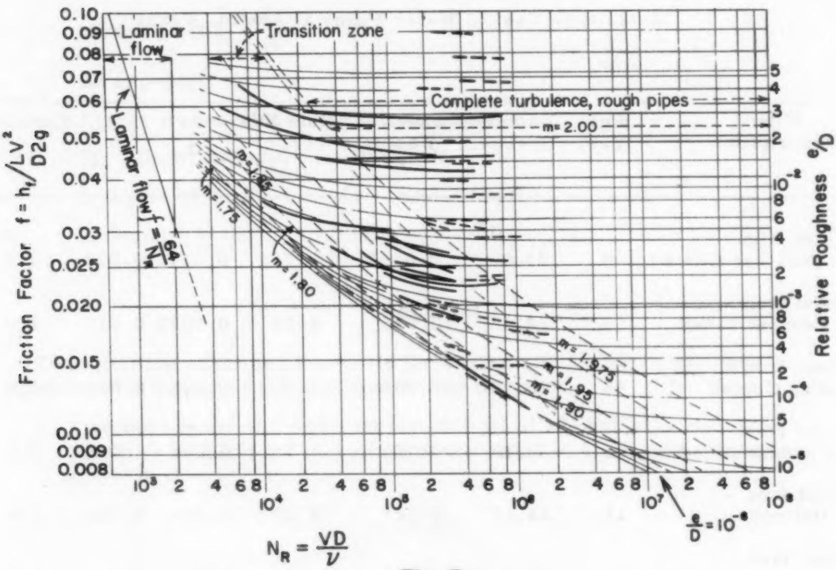


Fig. 3

Resistance Records on Old Pipes

C_o	Coefficient in relation between f and N_R
C_1	Coefficient in Hazen-Williams equation
D	Inside diameter of pipe - ft.
D_x	Inside Diameter of pipe - x inches.
f	Friction factor in Darcy-Weisbach equation.
g	Gravitational constant - ft/sec^2
h_f	Head loss due to pipe resistance - ft. of fluid
j	Summation index
K	Coefficient in equation $h_f = KQ^m$
K_o	Resistance factor a coefficient in equation $h_f = K_oQ^m$ chosen to be consistent with Moody diagram and Darcy-Weisbach equation

TABLE III

Resistance Test in Water Mains at Austin, Texas

Pipe Description	Age Yrs.	Diameter inches	Friction Factor f	Reynolds Number $x10^{-5}$	$\frac{e}{D}$	e ft.	Exponent m
Cast Iron - Coal Tar Coated	18	24.0	0.0467	5.75	0.017	0.034	2.0
Cast Iron - Cement Lined	3	24.12	0.035	6.10	0.0075	0.015	2.0
Cast Iron - Tar Coated	45	20.00	0.036	3.96	0.008	0.013	2.0
Cast Iron - Cement Lined	5	24.12	0.026	9.70	0.003	0.006	2.0
Cast Iron - Unlined	11	24.12	0.041	9.20	0.013	0.026	2.0
Cast Iron - Coal Tar Lined	27	12.2	0.078	4.14	0.06	0.061	2.0
Cast Iron - Cement Lined	9	6.0	0.021	1.36	0.001	0.0005	1.97
Cast Iron - Coal Tar Lined	18	8.21	0.082	2.29	0.065	0.045	2.0
Cast Iron - Cement Lined	6	20.12	0.020	7.65	0.0008	0.0014	1.97

K_o' **Equivalent resistance factor; K_{o_s}' for series pipes; K_{o_p}' for parallel pipes**

L **Length of pipe - ft.**

m **Exponent in equation $h_f = K_oQ^m$**

n **Roughness coefficient in Manning equation; number of pipes in series or parallel.**

Q **Volumetric rate of flow - cfs**

R **Hydraulic radius - ft.**

N_R Reynolds number $= \frac{VD}{\nu}$

S Slope of piezometric head line $= \frac{h_f}{L}$

V Mean velocity $= \frac{Q}{A}$ ft/sec

z An exponent

e Linear measure of absolute roughness - ft.

γ Specific weight of fluid - lbs/cu ft.

μ Dynamic viscosity of fluid - lbs sec/sq ft

ν Kinematic viscosity of fluid - sq ft/sec

APPENDIX

The following examples illustrate the steps in obtaining an exponential equation with the aid of the modified Moody diagram.

1. Find the exponential equation for 1000 ft. of 12-inch diameter steel pipe (e = 0.00015) for velocities of the order of 4.5 ft/sec with water at 60° F.

$$\frac{e}{D} = 0.00015;$$

$$N_R = \frac{4.5\ (1)}{1.21 \times 10^{-5}} = 3.7 \times 10^5.$$

From the modified Moody diagram f = 0.0155, m = 1.88.

$$h_f = f\ \frac{L}{D}\ \frac{V^2}{2g} = \frac{0.0155\ (1000)\ (4.5)^2}{(1)\ 2g} = 5.84 \text{ ft.}$$

$$Q = \frac{\pi}{4} D^2 V = \frac{\pi}{4} (1)\ (4.5) = 3.53 \text{ cfs}$$

$$K_o = \frac{h_f}{Q^m} = \frac{5.84}{3.53^{1.88}} = 0.544$$

$$h_f = 0.544\ Q^{1.88}$$

2. Find the exponential equation for 100 ft. of 6-inch cast iron pipe (e = 0.0009) for velocities of the order of 5 ft/sec with water at 60° F.

$$\frac{e}{D} = 0.0018$$

$$N_R = \frac{5\,(.5)}{1.21 \times 10^{-5}} = 2.06 \times 10^5$$

$$f = 0.0235; \quad m = 1.95$$

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} = 1.82 \text{ ft.}$$

$$Q = \frac{\pi}{4}\,(0.5)^2\,(5) = 0.98 \text{ cfs}$$

$$K_o = \frac{h_f}{Q^m} = \frac{1.82}{(0.98)^{1.95}} = 1.88$$

$$h_f = 1.88\,Q^{1.95}$$

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CONTENTS

DISCUSSION

DISCHARGE CHARACTERISTICS OF RECTANGULAR THIN-PLATE WEIRS[a]

Closure by Carl E. Kindsvater and Rolland W. Carter

CARL E. KINDSVATER,[1] M. ASCE and ROLLAND W. CARTER,[2] A. M. ASCE.—In concluding this discussion of a paper on the subject of weirs, it seems appropriate to observe that few subjects in the literature of hydraulics are more revealing of the personalities of its students. It is indeed remarkable that a comparatively simple physical phenomenon has inspired such an astounding number of printed words, formulas, laboratory investigations, and disagreements. Over a period of 250 years, nearly every name in the history of hydraulics is identified with contributions to the great store of weir literature. It is not amazing, therefore, that the most common ingredient in this inheritance is repetition.

Much more discouraging than the repetition which characterizes weir literature is the fact that some of the best thought on the subject is buried in the avalanche of printed words. On the other hand, the substantial area of ignorance regarding weirs which still exists is often dismissed with the casual implication that the solution to the problem is contained in a given formula —which is then quoted without reference to the qualifications which were prescribed by its author.

Pride and prejudice seem to have played a large part in the drama of the simple weir. Counter-claims of accuracy and scientific truth have been hurled across battle lines drawn up behind purely empirical formulas based on limited, often questionable experimental data. The futility of one such argument, which already occupies many pages in the Transactions, is revealed in Fig. 4(a). Here it is clearly demonstrated that, regardless of the quality of Bazin's data, the form of discharge equation used by Bazin and others is fundamentally inadequate.

Returning to the present, the writers are grateful for the constructive discussions of their paper which were submitted by Messrs. Sarpkaya, Kolupaila, Powell, Paull, Oki, and Carstens. It is regretted that several correspondents, including some respected European hydraulicians, were unable to submit their discussions within the time allotted for discussion.

The writers share with Mr. Sarpkaya his concern with the practical limitations of empirical equations. They are aware of the dangers of extrapolating empirical relationships beyond the range of conditions represented by the

a. Proc. Paper 1453, December, 1957, by Carl E. Kindsvater and Rolland W. Carter.

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experiments from which they were derived. Persons who publish empirical equations are obliged to state very clearly the purpose and limitations of their work. It is the reader's obligation to observe these limitations in the application of the results.

The writers have made it quite clear that they were concerned with the weir as a discharge-measuring device, and that, for practical reasons, their verification of the proposed formula was limited to "water at ordinary temperatures". Furthermore, the range of values of each variable involved in the experimental verification is explicit in the original paper. Without apologizing for the limitations which time and money placed on their work, the writers are, like Mr. Sarpkaya, hopeful that others will extend its boundaries. They are inclined to observe that, apart from the acknowledged limitations in their study of the fluid-property variables, their work covers a broader range of geometric conditions than any other single work known.

It should be quite clear from the original paper that values of k_h and k_b are dimensional quantities derived from experiments with a single liquid and a practical range of geometric conditions. The general relationships between k_h, k_b, R and W are carefully detailed. One accustomed to using the Reynolds and Weber numbers to describe the relative influence of viscosity and surface tension, respectively, would not expect "glucose, honey and molasses" to exhibit unique characteristics as long as they behave as true fluids.

Mr. Sarpkaya's Eq. (3) is a general statement of the slope-intercept form of the C_e curves (writers' Fig. 10), which are believed to be descriptive of the interrelationship of the principal geometric variables. His Eq. (2), on the other hand, involves a dimensionless representation of the fluid-property adjustment factors, k_h and k_b. It is not clear why he presumes that an additional fluid-property adjustment should be made to the relationship between the geometric ratios. Indeed, Eq. (5) appears to hold little promise of simplification or clarification. And, while it is considerably more complex than the writers' equation of discharge (Eq. (11), it is certainly no less empirical. Subsequent algebraic manipulations and the unlikely assumption that k_h is independent of surface tension do not lend credence to Mr. Sarpkaya's proposed equation.

Mr. Kolupaila incorrectly attributes certain objectives to the writers (e.g., "a search for higher precision [than ± 2%] was the purpose of the new investigation") and their sponsors (e.g., "The sponsor . . . expected a . . . formula correct to ± 5%") He is also incorrect in attributing to the writers' thoughts which are not to be found expressed in the original paper (e.g., "Their opinion is based on usually assumed values of α of about 1", and "the conclusion of the Walchensee tests was expressed as being unfavorable by the authors").

The writers agree with Mr. Kolupaila that the flow pattern for rectangular weirs might be subject to complete mathematical analysis if "a certain law of streamlines" were known. Unfortunately, the solution is hardly forthcoming in view of his subsequent qualification that such a law remains to be "established or approximation assumed" along with other "small correction factors". Mr. Kolupaila's remarks concerning k_1 and α are somewhat contradictory, but, in general, they appear to substantiate the writers' conclusions regarding the tenuous relationship between these two coefficients.

Mr. Kolupaila makes reference to many foreign publications on weirs, most of which had been read and evaluated by the writers. Failure to refer to certain of these publications in the original paper is evidence that they were not considered of particular value. It is not clear why Mr. Kolupaila refers

to the foreign publications without identifying and supporting his own preference among the diverse opinions and formulas proposed therein. A casual review of recent European reference works indicates that few of the papers referred to have been recognized as substantial contributions in their own homelands. Finally, an examination of the paper which describes the Walchensee tests will show that Mr. Kolupaila, not the writers, must have misinterpreted the conclusions drawn by Kirschmer and Esterer. The following free translation of their conclusions clearly supports the conclusions stated by the writers:

> "The results of the weir measurements substantiate the criticism of this method which has been expressed by others. Indeed, general reliance upon one of the rational formulae has not yet been satisfactorily proven".

Mr. Powell emphasized the unreliability of coefficients derived from experiments on physically small values of P. The writers have acknowledged this uncertainty as well as the uncertainty of the results of tests made with very small values of b and h in their investigation.

Mr. Powell's suggested method of correcting for the distribution of velocity in the approach channel is a very worthwhile and practical contribution. The results which he obtained from an analysis of the Schoder and Turner data are encouraging. It is hoped that additional tests made to define the "r" function will refine the method. Thus, the weir which is unavoidably located in a channel which produces a non-uniform velocity distribution might still be salvaged as a satisfactory meter. As pointed out by Mr. Powell, the ratio $r = V_a/V_b$ is essentially a characteristic of the installation which, like the dimensions of the weir, need be determined only once. It follows that his method of adjusting the coefficient of discharge is completely practical. It seems reasonable to assume that r is a function of h alone. Mr. Powell's willingness to settle for "errors of the order of one percent" is quite realistic, if not optimistic.

The writers appreciate Mr. Paull's kind remarks. They regret that they cannot reciprocate with an offering of dependable information regarding the inaccuracies to be expected under a full variety of applications. Disregarding such very real problems as variations in velocity distribution and "non-standard" geometric conditions, the order of magnitude of the errors to be expected are suggested by the scatter of the plotted points on the graphs showing the writers' data.

Mr. Oki brings to the discussion some new experimental data and formulas previously unknown to the writers. They are pleased to acknowledge his interest and criticism. In his Fig. 1 Mr. Oki proposes to demonstrate a relationship between C and h/b. In their original paper the writers state, "The independent influence of this ratio (b/h) is believed to be negligible over the full practical range of the other variables. An earlier investigation at the Georgia Institute of Technology supports this conclusion. . . . A few recorded efforts to incorporate the b/h ratio in discharge formulas are believed to be based on misinterpretations of influences related separately to the magnitudes of b and h."

The writers believe that this statement is actually supported by Mr. Oki's Fig. 1. Indeed, closer study will reveal that the slope of the curves on the figure shows the effect of b, whereas the vertical distance between the curves shows the effect of h. In fact, all of the data on Fig. 1 can be replotted to

show a relationship between C_d ($= C \div 2/3\sqrt{2g}$) and h for various constant values of b. However, if this is done, Mr. Oki's data appear to show a considerably larger influence due to both h and b than has been reported by the writers and others.

The writers are especially puzzled by some of the very low values of C_d shown on Mr. Oki's Fig. 1. All values of C_d for h/b greater than 0.5, for example, are less than the "theoretical" minimum, 0.611. This trend is believed to be unique in the literature. It is implied, of course, that the decrease in C_d is related to the increase in h/b. The writers can only repeat the conclusion, based on their own work, that the h/b ratio is not believed to be significant. In support of this conclusion they submit the observation that the data presented in the original paper cover a range of values of h/b from 0.03 to 6.0.

Mr. Carstens lists several reasons for the difference in the results obtained by applying the writers' method of analysis to the experimental data obtained by different investigators. The writers agree with his general diagnosis, but they fail to see the logic in his conclusion that the maximum experimental error in one set of data is an explanation of the difference between curves which were drawn to represent mean values for different sets of data. Such an unusual conclusion ignores, for example, the systematic variation with velocity distribution, weir crest condition, etc., shown by Schoder and Turner and others. To make their own position clear, the writers go on record as pointing to Mr. Carstens' item (b), namely, the "geometry of the experiments", as the major cause of the difference between the results of different experiments. A large part of the tremendous store of weir literature is evidence to support this conclusion.

Mr. Carstens' light dismissal of the "small differences" between various formulas ignores the purpose of most of the work done in this field since the original derivation of Eq. (15) by Weisbach. His comparison of the "theoretical" and measured values of C_d is a classic demonstration of circumstantial evidence rather than theoretical proof of the merit of the orifice analogy. The writers are pleased to endorse his conclusion that too many people write empirical equations (or equations that are too empirical!), but they fail to see the relationship between this popular thesis and the statement quoted from the original paper which preceded his pronouncement. They are also moved to observe that the weir which is the subject of Mr. Carstens' attention (i.e., $b/B = 1.0$) is only a limiting case of the problem under investigation, that values of h/P greater than 2.0 are impractical for reasonably accurate flow measurement, and that the fluid-property effects associated with small values of h and b cannot be ignored for all practical purposes.

In general, the discussers did not challenge the writers' candid disrespect for the traditional "theoretical" equation and the orifice analogy. However, most of them appeared to favor the use of a coefficient C_d which is equal to the writers' C divided by a quantity ($2/3\sqrt{2g}$) derived from the orifice analogy. The principal advantage attributed to C_d is that it is dimensionless.

The writers are staunch proponents of dimensionless ratios. A review of their publications will substantiate this statement. As engineers as well as researchers, however, they strive to be practical. In the present instance they question the logic of straining to make the coefficient of discharge dimensionless when the equation of discharge contains other terms which are dimensional. In the writers' equation, for example, evaluation of the discharge depends on knowledge of k_h and k_b, quantities which are absolute measures of

the combined influences of viscosity and surface tension. Similarly, in Eqs. (18) (Bazin), (19), (Rehbock), and (20) (S.I.A.), dimensional terms which are related to the magnitude of h restrict the application of those equations to water at ordinary temperatures.

It has been observed that the quantity $2/3\sqrt{2g}$ is obtained from the derivation based on the orifice analogy. Bazin and others have at one time proposed equations in which $2/3\sqrt{2g}$ was replaced by $\sqrt{2g}$ or $\sqrt{g}$. In these alternate forms of the discharge equation the coefficient is quite dimensionless, but the writers are loath to use them because of the almost exclusive division of preference between C (widely preferred by engineers in practice) and C_d (preferred by some researchers) as defined herein. While they find it difficult to get excited about the choice, the writers decided to use C mainly because it concedes to the practicing (American) engineer the margin of convenience.

The writers freely admit the advantage of C_c (rather than C_d. See Eq. (15)) for certain purposes. A reason seldom mentioned, for example, is that C_c is remarkably similar for different shapes of notch weirs. Thus, the researcher finds it convenient to use C_c to correlate meagre data from different sources. However, this reason does not justify the preservation of the C_c form of the discharge equation for practical use. It is hoped that this brief discussion of the relatively insignificant difference of opinion regarding C, C_d and C_c does not detract from the purpose of the original paper.

Corrections.—On page 5 of Proc. Paper 1453, in the third line following Eq. (6), the term $\sqrt{\sigma}/\rho$ should be changed to $\sqrt{\sigma/\rho}$. On page 27, in the first line of the paragraph containing Eq. (22), the word "new" should be changed to "few".

On page 1690-24, line 5, there should be no parenthesis after the 3.23.

On page 1690-25, eighth line below Eq. (F), "which Eq. (D)" should be "while Eq. (D)".

The term $\sqrt{2g}$ in the third line of the fourth paragraph on page 1690-22 should be $2/3\sqrt{2g}$.

THE TOTAL SEDIMENT LOAD OF STREAMS[a]

Closure by Emmett M. Laursen

EMMETT M. LAURSEN,[1] A. M. ASCE.—At first reading the three ways of solving the problem of total sediment load enumerated by Messrs. Garde and Albertson seem wise, considered and reasonable. Repeated reading, however, leads one to doubt, and finally reject, most of what is said or implied in this portion of their discussion. Not enough is known about fluid flow to theoretically describe in detail the state of affairs at a complexly rough fixed boundary. How then does one even begin a "theoretical" evaluation of the total sediment transportation rate? The total load is undoubtedly the sum of the suspended and the bed load but which bed load equation does one choose and how does one evaluate the reference concentration of the suspended load except by arbitrary assumption? (For a detailed discussion of the Einstein procedure and the modified Einstein procedure the reader may be interested in Reference (6)). This leaves dimensional analysis and intuition as possible methods. Dimensional analysis can only lead to the grouping of chosen variables. Ergo, intuition must reign supreme. Unfortunately, this state of affairs is all too true. Yet intuition is not entirely free, but must be bound by what is known.

As presented in the discussion, the relationship for the total sediment load put forward by Messrs. Garde and Albertson seems to have been developed mostly by intuition using dimensional analysis to form the dimensionless parameters and plotting and curve fitting to obtain the relationship between the parameters. In the form presented it appears to be different from other sediment transport equations. However, its similarity to most other equations becomes obvious if it is rewritten. Using the symbols of the original paper ($D = y_0$, $V_* = \sqrt{\tau_0/\rho}$)

$$C_t = \left(\frac{1}{\nu\sqrt{\rho}}\right)^3 (d\psi)^{9/2} \frac{\tau_o^{3/2}}{y_o^{3/2}}$$

Note that ψ is a function of d alone, and the curve on the log-log plot can be approximated by the straight line $\psi = 0.12\, d^{-3/2}$. This equation can then be further transformed through the use of the relation q_s = const. $c_t\, q$ and the Manning equation to read

$$q_s = C \frac{n^3}{d^{9/4}} \frac{V^4}{y_0}$$

where the coefficient C has been allowed to absorb the various factors which are constant or almost so. Similarly transformed, and with the critical term dropped, other equations are very similar.(1)

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a. Proc. Paper 1530, February, 1958, by Emmett M. Laursen.

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DuBoys (Straub) $$q_s = C_1 n^4 \frac{V^4}{y_o^{2/3}}$$

Schoklitch $$q_s = C_2 \frac{n^3}{d^{1/2}} \frac{V^4}{y_o}$$

Meyer-Peter $$q_s = C_3 n^3 \frac{V^4}{y_o}$$

WES $$q_s = C_4 n^{2m-1} \frac{V^{2m}}{y_o^{m/3}}$$

Shields $$q_s = C_5 \frac{n^4}{d} \frac{V^5}{y_o^{2/3}}$$

Brown-Einstein $$q_s = C_6 \frac{n^6}{d^{3/2}} \frac{V^6}{y_o}$$

Brown-Kalinske $$q_s = C_7 \frac{n^5}{d} \frac{V^5}{y_o^{5/6}}$$

Note that in some of these equations a part of the effect of the sediment size is contained in the coefficient.

Mr. Bogardi's interesting contribution, like that of Messrs. Garde and Albertson, is more a reporting of similar work than a discussion of the concepts of the original paper. The relationships presented delineating the bed configuration could be very useful if confirmed over a wider range of conditions. The writer hopes that in Mr. Bogardi's projected paper, the rationale of at least the parameters, and preferably also the relations, will be discussed at greater length. The use of the total tractive force (or the shear derived therefrom) in the bed stability factor and the diameter of the sediment by itself seem to be questionable procedures.

The discussion of Mr. Bondurant is of the type that the writer had most hoped for—the application of the proposed relationships to field measurements. Considering the change in scale and general environment between laboratory flumes, which provided the data upon which the relationships were built, and the Missouri and Arkansas rivers, the factor of four, or the shift in the $f(U_*/w)$ curve, is not too disheartening. The discrepancy could be due either to this gross change in scale or to deficiencies in the proposed relationships, or possibly to both reasons. The fact that the original curve described the Niobrara measurements so well and that the shift of the curve for the Missouri river data from Omaha was less than for the data at Kansas City would indicate that some factor such as the Reynolds number might be useful in collapsing the different results. More field measurements might permit the development of such a stop-gap modification and thereby enhance the usefulness of the proposed relationships.

WATER DISTRIBUTION DESIGN AND THE McILROY NETWORK ANALYZER[a]

Closure by M. B. McPherson and J. V. Radziul

M. B. McPHERSON,[1] A. M. ASCE and J. V. RADZIUL.[2]—The supplementary experiences, observations and comments contributed by Messrs. Graves and Branscome, Mr. Cole, and Mr. Lomax,[3] are sincerely appreciated.

The need expressed in the paper for publication of a proven standard set of instructions for water distribution network analysis for a stored-program type of computer has been satisfied by Messrs. Graves and Branscome.(1-a) Their paper and the authors' appeared in the Proceedings simultaneously. Subsequently, Mr. Hamblen(1-c) offered a program suited to calculations for larger networks. With the publication of these programs there is no longer a valid excuse for rough approximation by means of extensive shortcuts in the design or investigation of complex water distribution networks by consulting engineers and utilities simply because a McIlroy Analyzer does not happen to be readily available to them.

Mr. Lomax says, "The possibilities of the digital computer and the analyzer have not been fully developed," and illustrates some of these potentials by itemizing several unusual problems he has analyzed. The authors are in full agreement with his statement: "The combination of the analyzer and the computer (at his institution) will provide complementary rather than competitive services," but also foresee valuable collateral uses for both devices in distribution system analysis by the larger utilities.

Since large networks are almost always designed on the basis of peak day conditions, "computations on the basis of the largest rates of demand" (Graves and Branscome) are normally needed regardless of the device used in performing the computations. Without equalizing storage, the demand of the peak hour of the maximum day normally constitutes the critical design condition for large networks. With storage, the minimum hour of the maximum day demand is apt to be more critical because the storage is necessarily being replenished at a high rate at that time and because the flow to storage normally traverses a long distance across the network. The design of arterial piping improvements is a trial and error procedure. When estimated future demands (always taken at rates greater than current demands) are being studied, the authors use an initial arterial network which is usually a moderate modification of existing piping but which will probably yield excessive head losses. They have been able to reach a more satisfactory final piping

a. Proc. Paper 1588, April, 1958, by M. B. McPherson and J. V. Radziul.
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3. Discussions appeared in Proc. Paper 1856, p. 85-88, Nov., 1958.

arrangement more quickly in this manner than by starting with a network which would undoubtedly prove to be more than adequate. Hence the statement in the paper: "--- needed system revisions are evident at inflow rates well below the design rates; obviously needed piping improvements are then made prior to any formal or complete test runs."

Messrs. Graves and Branscome correctly note, with reference to Fig. 6 of the paper, that the statement: "A revised storage rate requires a new storage curve," is not clear. The balancing of demands, storage rates and near-uniform pumping rates is difficult to describe without illustration. However, a detailed example of balancing procedures using hour-by-hour analyses for the City of Philadelphia's Belmont High Service District, with a single pumping station and one storage site, is to be published soon.(2)

Mr. Cole raises the question of economic justification of the equalizing storage required for near-uniform pumping versus the lesser storage needed for stepped pumping. Comparisons are extremely complicated and no universal rule can be applied even between service districts of the same water works. The principal benefit sought in the installation of equalizing storage is the provision of more stable pressures and less service interruption than can be achieved by direct pumping. Unfortunately, the storage volumes needed to secure near-uniform pumping, augmentation of pumping station output during fires, and outage relief, are neither a summation of the volumes needed for each purpose nor a readily determined overlapping of volumes.

Consider first, peak day storage. As an example, if the maximum day demand is about 150% of the annual average, 10% of the average day demand is needed for near-uniform pumping on the average day and 10% of the maximum day demand is needed for near-uniform pumping on the maximum day, the storage volume required for the maximum day is thus about 1.5 times the volume required for the average day. This theoretical ratio ranges between 1.3 and 2.1 for the five districts cited in Fig. 4. The ratio is undoubtedly higher in many systems across the country. Maximum day storage raises and stabilizes critical low pressures under non-emergency operating conditions. Benefits received on days of lesser demand might be regarded as bonuses not chargeable to the cost of the maximum day storage. If the storage provided is not sufficient to meet peak day variations, the resultant higher pumping rates on peak days will generally require a more expensive distribution network than would be satisfactory for near-uniform pumping with more storage.

At what period during the design maximum day should fire requirements be met? Among other things, the N.B.F.U. rates a district on ability to deliver the required fire flow during a district demand equal to the maximum day rate. Philadelphia's design policy is to provide a working elevated storage volume (upper 25-feet) adequate to meet future maximum day demands under hypothetically perfect uniform pumping (exceeds somewhat the quantity needed for near-uniform pumping). An additional, separate increment of storage for fire flow augmentation is not provided. The fire flow increment is to be delivered from the pumping station. Special design analyses are made to insure adequate fire pressures under these conditions. In large districts, flow and pressure requirements for fires seldom dictate additions over and above the piping needed to satisfactorily meet peak day demands.

For what duration and for what hour of the day should outage relief be provided? In Philadelphia it is not considered economically feasible to earmark a special storage increment, or special piping, for outage relief alone.

However, the extra benefit of added outage relief has been a deciding factor in adopting some otherwise marginally justified, needed improvements.

Unless the total equalizing storage volume provided is at least sufficient to meet average day demand variations, the benefits which will be derived from the storage on a peak day will be almost inconsequential. Therefore, the minimum amount of storage worthy of serious consideration should be the volume required to meet average day demand variations. With this minimum, near-uniform pumping may often be feasible for more than half of the year's demands, as inferred in Fig. 3. The annual cost of pumping will be greatly influenced by the large share of demands thus met by near-uniform pumping with a single pump. The cost of an increment of storage above the minimum to meet peak day fluctuations can be readily compared against the cost of additional piping which would be required to provide similar service pressures by means of direct peak day pumping without storage. The more important comparison, from a cost standpoint, between the use of minimum storage with stepped pumping on the peak day and either direct or near-uniform pumping would be purely speculative. For stepped pumping, it is almost impossible to realistically predict the most probable pattern of peak day demands and the sequence and timing most likely to be followed by the station operator in meeting these variations.

The dollar value of the more reliable overall service afforded by equalizing storage is largely intangible despite its obvious importance. Unfortunately, the cost of an increment of storage especially set aside to more adequately meet fire demands cannot be equated in direct terms to a specific expected reduction in fire insurance policy rates.

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   c. Discussion by J. W. Hamblen, and authors' closure, ASCE Proc. Paper 1855, pp. 11-17, November, 1958.

2. McPherson, M. B. and Radziul, J. V., "Distribution Design Considerations," Jour. A.W.W.A., pending, January or February, 1959.

SYNTHETIC FLOOD FREQUENCY[a]

Discussions by Ven Te Chow and J. L. H. Paulhus

VEN TE CHOW,[1] A. M. ASCE.—The author has presented an interesting procedure for synthesizing flood frequencies through the use of a rational formula and unit hydrograph. In essence, the lengthy derivation of the procedure may be boiled down to the effect that the flood discharge can be computed by the conventional rational formula and then reduced in the magnitude in order to convert the value for the rainfall frequency to that for the runoff of the same frequency. When the conventional rational formula is used, it is logical to accept that the frequency of the computed discharge is compatible with the frequency of the rainfall intensity which is used in the formula. Owing to the detention and retention characteristics of a watershed, the actual frequency of the resultant discharge is in general much higher than the frequency of the corresponding rainfall. If the computed discharge takes the same frequency of the rainfall, the computed value should be therefore reduced.

The above discussion may be expressed by a modified rational formula:

$$Q = KCIA \tag{14}$$

where Q is the peak discharge in cfs., K is a reduction factor, C is a runoff coefficient, I is the rainfall intensity in in. per hr., and A is the drainage area in acres.

The use of the unit-hydrograph theory in the author's derivation indicates that the rational formula in the form of Eq. (10) is a special case of the unit-hydrograph method. A general mathematical proof of this point has been demonstrated by Nash.(1)

Eq. (14) is in fact given in the form of Eq. (13) in the paper. The author found that the value of K in both formulas may be considered as a constant, that is equal to 0.44 for overland flow on turf and equal to 0.77 for natural areas, sewered areas, and overland flow on pavements. In other words, the relationship between the rainfall and runoff for a given frequency is assumed constant. It is believed that the author has accepted that rains of the same duration and frequency occur with associated runoff producing conditions that vary from time to time depending on antecedent factors. It may be then reasoned that an averaged rainfall-runoff relationship for that group of rainfalls would produce an acceptable runoff volume-frequency value. The truthfulness of this reasoning can only be verified by the use of actual data. Although the author has shown several actual examples for this purpose, it

a. Proc. Paper 1808, October, 1958, by Franklin F. Snyder.
1. Prof. of Hydr. Eng., Univ. of Illinois, Urbana, Ill.

would be interesting to see how this reasoning would apply to many other areas which are different from those investigated by the author.

It is generally recognized that two major factors that lead to runoff are the storm rainfall, a climatic factor, and the antecedent conditions, a physiographic factor. The probability occurrences of rainfall and runoff are well-known, while the probability occurrence of the physiographic factor is a subject that has not yet been explored. From a probability viewpoint, however, the physiographic factor is a statistical variable just as the rainfall and runoff are statistic variables.[2,3] The frequency of runoff is the product of the frequencies of rainfall and physiographic condition. When an average relationship between rainfall and runoff frequencies is assumed, the frequency of physiographic variation is automatically taken as constant. Whether this is true or not remains to be seen from future verification by data of various geographical characteristics.

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3. Ven Te Chow, Discussion of Frequency of Discharges from Ungaged Catchments, Trans., Amer. Geophys. Union, Vol. 38, No. 6, pp. 963-966, December 1957.

J. L. H. PAULHUS,[1] M. ASCE.—A novel and generally logical approach to the flood-frequency problem has been developed by Mr. Snyder. His procedure appears suitable for deriving flood-frequency data for very small areas, especially if they have a large percentage of impervious surface, provided representative rainfall-frequency data are available. There are, however, several weak points in its development that make Snyder's approach less adaptable to larger basins. In general, when representative rainfall-frequency data are used, the weaknesses would indicate a tendency for the procedure to yield results somewhat higher than might reasonably be expected.

Perhaps the biggest drawback to the successful application of Snyder's procedure, even when properly developed, is the lack of means for selecting the most representative rainfall-frequency data to use. Snyder advises that variation in annual precipitation must be considered in the selection or adjustment of the point rainfall record to be used but omits details. This practice does not necessarily produce reliable results. Table 1 shows the different flood-frequency values that would be obtained for Wills Creek near Cumberland, Md., by applying Snyder's procedure to rainfall-frequency data, unadjusted except for partial-duration series, for the five nearest surrounding stations for which that type of data are already available. Pittsburgh and Elkins are each about 75 miles from Cumberland and the other three stations are between 105 and 115 miles away. The two nearest stations yield widely different sets of results, with flood-frequency values based on Elkins rainfall being about

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Table 1. FLOOD-FREQUENCY DATA (100CFS) FOR WILLS CREEK BASED ON 5 STATIONS

Return period (yr)	2	5	10	25	50	100
Obs. record, 27 yr	92	155	203	268	316	362
Pittsburgh, Penn.	57	78	96	119	138	159
Elkins, W. Va.	75	111	144	191	233	279
Harrisburg, Penn.	80	115	148	198	237	279
Baltimore, Md.	105	154	194	256	311	363
Washington, D. C.	125	178	225	290	358	424

50 per cent higher than those based on Pittsburgh. Which station is representative for the basin? Noting also that Baltimore yields results over 25 per cent higher than those of Harrisburg, with normal annual precipitation of 42.59 and 36.01 in., respectively, which of those two stations, or what kind of adjustment, would one use in developing a synthetic flood-frequency curve for a basin midway between the two cities?

The results of Table 1 would be changed very little by adjustment of basic rainfall-frequency data for differences in mean annual precipitation. Stations in and near Wills Creek Basin indicate a mean annual precipitation between Pittsburgh's and Elkins'. The differences would not provide a suitable basis for an adjustment yielding flood-frequency values approximating the observed. It is possible that neither Elkins nor Pittsburgh yields results comparable to the observed because Snyder's rainfall-runoff relation is not representative for Wills Creek. There is a very small chance that it could be considering its derivation, which is discussed later.

One of the more serious shortcomings of the procedure is the assumption that the rainfall intensity for a given return period will produce a peak-discharge value having the same return period. For example, the 10-year rainfall intensity is assumed to produce the 10-year flood. In this respect, Snyder's procedure is no better than formulas such as the so-called rational, or CiA, formula.

Annual maximum peak discharges often occur in connection with rainfall intensities less than the annual maximum intensities. The reason is that in many regions the highest intensities occur in the seasons when soil conditions are not generally favorable to high runoff rates. For example, Snyder would presumably use rainfall intensities for Washington, D. C., to develop flood-frequency data for the Rappahannock River near Fredericksburg, Va., which has a drainage area of 1599 square miles and a computed concentration time (T_c) of 24.8 hours if it is assumed that the impervious area of the basin amounts to less than 1 per cent. The rainfall-frequency data he would use were obtained by the Gumbel method of analysis utilizing the annual maximum values for the 57-year period 1896-97, 1899-1953. The results were then adjusted by Snyder for a partial duration series and converted from intensities to amounts to yield his Fig. 2, Appendix 1. Comparison of the monthly distribution of the annual floods of the Rappahannock for the total 45-year period (1913-57) of peak discharge record with that of the annual maximum 24-hour rainfall intensities used in the Gumbel analysis shows a relatively poor relationship (Table 2).

It is readily seen that the 3-month period yielding the greatest percentage of annual floods is March-May with 37.9 per cent. Only 15.8 per cent of the annual maximum 24-hour rainfalls occurred in the same period. On the other hand, while the 3-month period, July-September, yielded 57.8 per cent of the annual maximum 24-hour rainfalls, only 26.6 per cent of the annual floods occurred during this season.

Table 2. MONTHLY DISTRIBUTION (PERCENT) OF ANNUAL MAXIMUM FLOODS AND 24-HOUR RAINFALLS

Month	Ann. Floods (Rappahannock)	Ann. Max. 24-hr Rainfalls (Washington)
Jan.	6.7	0.0
Feb.	2.2	1.7
Mar.	8.9	3.5
Apr.	17.9	5.3
May	11.1	7.0
Jun.	4.4	8.8
Jul.	4.4	17.5
Aug.	11.1	15.8
Sep.	11.1	24.5
Oct.	8.9	5.3
Nov.	8.9	5.3
Dec.	4.4	5.3

Since the monthly distribution might reasonably be expected to be better than the above for a small basin for which point rainfall would be more representative, it appeared advisable to make a test. Unfortunately, there are no streamflow records of adequate length for any small basin in the area of interest. Consequently, the writer synthesized a record of annual floods for the Little Falls Branch near Bethesda, Md., drainage area 4.1 square miles, using the procedure outlined in Proc. Paper 1451.(1) Hourly rainfall data for Baltimore, Md., were used in synthesizing the annual flood record as they were already available to the writer.

Bearing in mind that the writer was, at this point, concerned only with the comparison of the monthly distribution of annual floods and annual maximum rainfall values, it should be noted that the accuracy of the 1-hour unit hydrograph used in synthesizing the annual flood record had little bearing on their monthly distribution. The reason for this is that the annual maximum 1-hour computed runoff was almost always responsible for the annual peak discharge for the basin, which, according to the author, has a computed T_c of 1.65 hours. Consequently, the comparison could have been made directly by using the monthly distribution of annual maximum hourly runoff with no appreciable change in the results.

Annual maximum peak discharges were synthesized for the 61-year period, 1894-1954, and their monthly distribution noted. The monthly distribution of annual maximum 2-hour rainfalls at Baltimore, Md., for the 49-year period, 1903-51, used in the Gumbel analysis for U. S. Weather Bureau Tech. Paper 25,(2) was then determined. It was noted that annual maximum 1-hour rainfalls had almost exactly the same monthly distribution as the 2-hour. The comparison of monthly distributions of annual maximum floods and 2-hour rainfalls is given in Table 3. As was expected, the latter comparison shows better agreement than the first. The 3-month period, July-September, has the greatest percentage of annual floods and annual maximum 2-hour rainfalls although the proportions differ considerably, 52.5 vs. 71.5 per cent, respectively. Actually, of the 49 values of observed annual maximum 2-hour rainfalls, 30, or 61 per cent, produced annual maximum synthesized peak discharges. The other 19 synthesized annual maximum peak discharges resulted from lesser rainfalls occurring with more favorable flood-producing soil conditions.

Table 3. MONTHLY DISTRIBUTION (PERCENT) OF ANNUAL MAXIMUM FLOODS AND 2-HOUR RAINFALLS

Month	Ann. Floods (Little Falls Br.)	Ann. Max. 2-hour Rainfalls (Baltimore, Md.)
Jan.	1.6	0.0
Feb.	1.6	0.0
Mar.	3.3	0.0
Apr.	6.6	2.0
May	11.5	6.1
Jun.	8.2	12.3
Jul.	18.0	28.6
Aug.	23.0	28.6
Sep.	11.5	14.3
Oct.	4.9	4.1
Nov.	4.9	2.0
Dec.	4.9	2.0

The above comparisons suggest that, other things being equal, Snyder's procedure should tend to yield flood-frequency values that are too high. Unfortunately, only one of the basins the author used has a streamflow record of sufficient length to yield a reliable test. The Rappahannock River near Fredericksburg, Va., has a 45-year record (1913-57) of annual peak discharges. The annual floods were analyzed by the Gumbel method and the results adjusted for a partial duration series. The comparison between these adjusted values and those to be obtained by Snyder's method are presented in Table 4.

Table 4. FLOOD-FREQUENCY DATA, RAPPAHANNOCK RIVER NEAR FREDERICKSBURG, VA., (100 CFS).

Return period (yrs)	2	5	10	25	50	100
Obs. 45-yr record	380	590	748	960	1120	1279
Snyder's method	384	560	720	968	1160	1360

In view of the author's expressed belief that a flood-synthesizing procedure such as described in Proc. Paper 1451(1) would yield more uncertain results than his method, comparisons were made for two other basins used by him and for which annual floods had already been synthesized. These two were Goose Creek near Leesburg, Va., drainage area 338 sq. mi., and Little Falls Branch near Bethesda, Md., drainage area 4.1 sq. mi. The observed and synthesized annual floods were analyzed by the Gumbel method and then adjusted for a partial duration series. The results are listed in Table 5 for comparison with the values obtained by Snyder's method:

Table 5. FLOOD-FREQUENCY DATA, GOOSE CREEK NEAR LEESBURG, VA., (100CFS).

Return period (yr)	2	5	10	25	50	100
Obs. 26-yr. record	126	233	308	410	485	562
Snyder's method	115	175	216	287	348	407
Synthesized floods, 55 yrs.	98	163	210	277	327	376

LITTLE FALLS BRANCH NEAR BETHESDA, MD., (10 CFS).

Return period (yr)	2	5	10	25	50	100
Obs. 13-yr. record	116	160	191	235	270	301
Snyder's method (Wash. data)	118	144	167	208	241	276
Snyder's method (Baltimore data)	90	122	152	189	222	255
Synthesized floods, 61 yrs.	100	133	158	191	217	241

The comparative tabulations of flood-frequency data in Tables 4 and 5 indicate that Snyder's procedure yields results in fairly good agreement with the frequency data obtained from the observed streamflow record for the Rappahannock and the synthesized floods for Goose Creek and Little Falls Branch. However, the fact that in all three comparisons Snyder's procedure yields the highest 50- and 100-year floods, whereas his values for other recurrence intervals may be lower or higher, suggests a possible bias in his method.

Incidentally, the writer was unable to determine how the author obtained the Gumbel values he listed for Goose Creek and Little Falls Branch. The values listed above in Table 5 can be expected to be significantly higher than the author's for recurrence intervals of 10 years and under because they have been adjusted for a partial duration series. However, the differences are such that they cannot be explained by that alone. In both cases, the author's computations appear to be at fault. In the case of Goose Creek, two annual peak discharges of 45,000 cfs and one of 32,800 cfs during the 26-year period of record indicate that the author's values are much too low. In the case of Little Falls Branch, the writer used an additional year of record but that value was the minimum annual peak discharge of record. Consequently, there should be no appreciable difference between the frequency values from the 12-year record and those from the 13-year. However, the values presented by the author in his Table 2B are again appreciably lower than the writer's (Table 5). Possible explanations are that the author obtained his Gumbel data by fitting a line visually instead of by computation and/or that he did not consider published and unpublished revisions of published annual peak discharges.

Perhaps the weakest link in the development of Snyder's procedure is the rainfall-runoff relation, which is based on monthly precipitation and runoff data. He assumes that such a relation tends to represent storm situations and advances questionable reasons. Nevertheless, he subjectively drew his curve to yield "slightly higher" runoff than would be indicated by a mean curve. Why? How much higher? One is inclined to conclude that the curve had to be drawn to yield some pre-determined results.

The use of a single curve to represent even average storm rainfall-runoff situations is not sound. For example, his relation (Fig. 3, App. 1) indicates about 48 per cent runoff for 5 inches of rainfall. Presumably, it makes no difference whether the 5-inch rainfall falls in one hour or 24 hours. This is, of course, contrary to fact and therefore unacceptable. A duration, or time of concentration, parameter must be included in the relation.

In order to determine how well Snyder's relation fitted individual basins, relations based on monthly precipitation-runoff data were derived for five of the basins listed by him. Fig. A shows the relations for the smallest and largest natural basins, namely, Little Falls Branch and Rappahannock River, with Snyder's relation superimposed. In both cases, as in the other cases not shown, a seasonal effect is noted. In the case of Little Falls Branch, Snyder's relation appears to yield considerably higher than a mean curve for all the

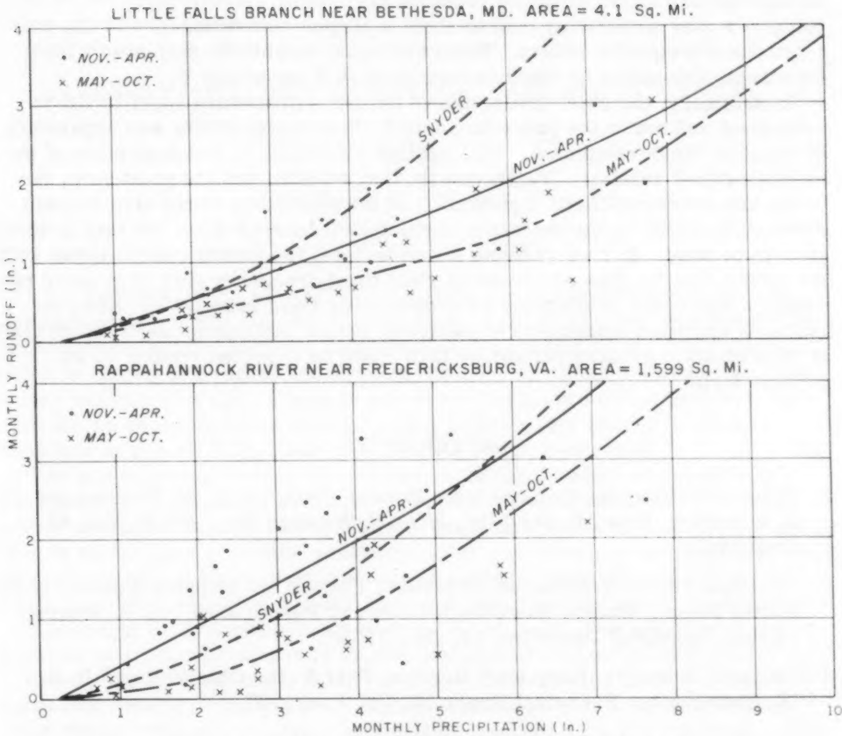

Figure A. MONTHLY RAINFALL-RUNOFF RELATIONS

data would indicate. Fig. A suggests that Snyder's mean relation does not properly represent mean monthly precipitation-runoff relations, let alone storm rainfall-runoff conditions, for some individual basins in the area of interest. In fact, the indications are that there is a very small chance that his relation would approximate average storm-runoff conditions on any basin.

Another weak step in the development of Snyder's procedure is the point-area relation. He assumed a random distribution of storm centers over a circular basin and obtained two sets of reduction factors, one for summer storms and one for winter storms. The former was assumed to apply to concentration times of 4 hours and less and the latter to 6 hours and more. Since frequency is involved, it would have been much better to have used the point-area relation presented in U. S. Weather Bureau Technical Paper 29,[3] to which the author refers. That relation is based on a large amount of observed data of comparable frequencies from unusually dense precipitation networks and is therefore most applicable to Snyder's problem.

As implied above, considerable subjectivity may be involved in the selection of the proper point rainfall-frequency data for the computation of flood frequencies for a particular basin. Subjectivity may also be involved in the selection of a representative value of the friction factor, n, and in estimating the percentage of area that is impervious or drained by sewers and the

percentage of natural drainage channels eliminated. A difference of .01 in the value of n may sometimes lead to about a 10-per cent difference in the computed flood-frequency values. Errors of equal magnitude may result from incorrect evaluations of other factors used in determining T_c.

In summary, the chief criticisms of Snyder's procedure apply to its development and not to the procedure itself. Some subjectivity was apparently involved in the development. This applies especially to the derivation of the rainfall-runoff relation. Furthermore, that relation and the point-area relation are oversimplified. Application of the procedure would also require some subjectivity in the selection of rainfall-frequency data, friction factors, etc., to be used. In view of these shortcomings, the writer cannot agree with the author that his procedure would yield flood-frequency data of greater reliability than could be obtained by synthesizing flood records.(1) However, Snyder's approach appears to be basically sound, and can be modified to yield results of much greater reliability than could be expected from it in its present form.

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2. "Rainfall Intensity-Duration-Frequency Curves for Selected Stations in the United States, Alaska, Hawaiian Islands and Puerto Rico," U. S. Weather Bureau Technical Paper No. 25, Dec. 1955.

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DIVINING RODS VERSUS HYDROLOGIC DATA AND RESEARCH[a]

Discussion by Ven Te Chow

VEN TE CHOW,[1] A. M. ASCE.—The author has presented an extremely stimulating paper on the history, philosophy, and the general situation of the hydrologic data collection and treatment at the present time. This presentation is timely and valuable because hydrology as either a science or a technology has come of age. Owing to a general negligence of the problem, the need for an adequate basic-data program in hydrology should be greatly emphasized and must be made known to all people concerned.

The use of diving rods in the discussion of the problem in question is interesting and appropriate. In fact, the philosophy of divining rods is not unique in the development of hydrologic problems, while its essence is also true in many other scientific problems. The acceptance of witching philosophy by the majority of the society constitutes indeed a stumbling-block to the progress of technology and to the development of any scientific program.

According to a recent survey made in the study at Harvard University, a great number of approximately 25,000 diviners are known in existence, currently plying their trade in the U. S. A.[2] The conclusion of this study indicates that "waterwitching will persist in this country as long as individuals must choose well sites under conditions where they have few external guides as to which of several possible sites will minimize their chances of paying heavily for the water they need." A modern basic-data program for collecting and interpreting hydrologic information should supply sufficient "external guides" to those who are in need and therefore will end any attempt of a mystical decision-maker.

The gap between the data collection and the data processing and interpretating is apparent from the author's statement: "About 20,000 pages of water data are published annually. Only a few hundred pages are devoted to interpretive and research results." It is economically unjustifiable that much effort and hence money have been spent on data collection while relatively little interpretive results are produced. Although many agencies are engaging in the collection and publication of hydrologic data, there is seemly lacking in a concerted effort or unified attempt to make the information readily available to a researcher in the field. Many times the researcher has to spend much energy in collecting the information from the information collected by different agencies. Therefore, one thing that a basic-data program should contain or a unified hydrologic-data agency, if proved to be feasible, should do is to

a. Proc. Paper 1809, October, 1958, by W. B. Langbein.
1. Prof. of Hydr. Eng., Univ. of Illinois, Urbana, Ill.
2. Ray Hyman and Evon Z. Vogt, 1958. Some Facts and Theories on Water-Witching in the United States: GeoTimes II (9), March 1958, pp. 6-7 and 19.

classify and collect the information from the individual collections made by all public and private agencies. Such classification and collection can be easily recorded on IBM cards and an index to the information should be issued from time to time. The U. S. Geological Survey is currently publishing indexes of surface-water records in the United States. Such indexes should be expanded and extended to cover all kinds of hydrologic data by a sort of centralized agency.

WAVE FORCES ON SUBMERGED STRUCTURES[a]

Discussion by Turgut Sarpkaya

TURGUT SARPKAYA,[1] A. M. ASCE.—The timeliness of this paper lies in the fact that while it provides design data much needed for the application of analytical methods in the determination of forces on some types of submerged structures, it shows the method of handling the elusive problem of determining the drag in unsteady flow by means of an admittedly somewhat unrealistic but quite simple analysis.

This writer's comments and criticisms will be on testing facilities, test procedure, and on the basic concept of virtual mass and drag in unsteady flow.

1. Under the heading "Test procedure", the authors state that "For certain wave frequencies, standing transverse waves were developed which produced a variation in wave height from one side of the model to the other as much as 40 per cent. For such conditions, a single measurement of wave height at the center of the model provided a value which differed as much as five per cent from the average effective wave height. This phenomenon was caused by the establishment of a resonant motion in the channel and tank, and no method was found to eliminate it." Basically the problem is that of the transversal instability of oscillatory gravity waves in open channels. Suquet and Wallet(1) and Milne-Thomson(2) have reported similar instabilities. Gawn(3) has demonstrated experimentally that when the frequency of the oscillations of the cylindric plunger exceeds a certain well defined critical value, transverse waves are also set up resulting in a three dimensional gravitational oscillation. In addition to the resonant motion set up in the model basin as described by the authors, the presence of vertical walls of the channel and most important of all the particular wave generating mechanism is believed to be responsible for the transversal instability of waves. This belief is supported by the fact that a series of plates attached perpendicularly to the plunger at regular intervals have prevented or eliminated the transversal standing waves.(1)

If the authors have employed such anti-standing wave plates on the plunger, wave filters immediately at the downstream side of the plunger, and artificial wave absorbers at the end of the test channel, they might have prevented the formation of standing transverse waves.

2. This writer believes that the authors are well justified in using Airy's equation for oscillatory gravity waves, rather than those of Stokes or Struik, for the reason that the experimental results reported(4,5) are in better

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a. Proc. Paper 1833, November, 1958, by Ernest F. Brater, John S. McNown, and Leslie D. Stair.

1. Asst. Prof., Eng. Mechanics Dept., Univ. of Nebraska, Lincoln, Nebr.

agreement with Airy's equation as far as the velocity of waves are concerned. However, it should be noted that the wave profile is described better by Stokes' or Struik's equations. The deviation of a sinusoidal curve from the actual profile might slightly effect the ΔH value.

3. Computation of oscillatory wave forces or the sum of the drag and the inertial force on the submerged structures under consideration is based upon the assumption that the flow pattern is not disturbed by the presence of the model. For the flat plates the values of C_d and C_m were selected to give the best correspondence with the measured values of phase angles and the magnitudes of the maximum forces. For barge models the values of phase angles and the magnitudes of the maximum forces. For barge models the values of C_m were computed using the measured values of inertial forces. Hence, for the barges there is no way of knowing how approximate the values of C_m are, and how much justification there is for the simplified analysis. One can only say that C_m is assumed to take into account whatever discrepancies may arise between a more realistic analysis and a simplified one. For rectangular barge models the authors found a C_m value which varied from 1.31 to 1.76. At first these values seem to compare favorably with the theoretical value of C_m obtained from a two-dimensional flow analysis ignoring the effects of the time dependent wake surrounding the body. If, however, one prepares a plot of the rate of change of C_m with respect to the ratio (a/b), it is easy to observe that for the barge model tested the value of C_m does not vary more than a few per cent if only the width of the model is increased by 50%, i.e., from 10 in., to 15 inches. Hence, a wake that would form only behind the plate would not have any appreciable effect on the value of C_m. Since, however, the sharp edges lead to separation for very small relative motions, resulting vortices grow into a wake and extend the disturbed flow region also laterally as well as transversely. Therefore, in comparing the experimental C_m values with the ones obtained from a two dimensional flow analysis, one should take into consideration the equivalent potential flow model which is supposed to be made of a surface enclosing both the body and its time dependent wake..

In order to determine the effect of the three dimensional character of the models on the value of C_m, plexiglass models (1/2 geometrical ratio) of the right rectangular prism and the slotted barges shown in the authors' Fig. 4, were constructed in the hydrodynamics laboratory of the Engineering Mechanics department of the University of Nebraska, and the C values (added mass/displaced mass) were determined by means of vibratory motion. For a full rectangular barge the value was of 0.35, and for the slotted barge 0.65. Corresponding C_m values are 1.35, and 1.65, respectively. In the tests at Nebraska, the velocities varied from 0.2 to 0.6 in/sec., the frequencies from 20 to 60 cycles per second, and the accelerations from 40 to 400 in/sec.sec. The above values of C_m will be used later in computing the reduction in horizontal forces on slotted barges.

In what follows, this writer would like to review the recent ideas concerning virtual mass and then point out the parameters that cause marked changes in both C_d and C_m. Although, considerable progress has been made on the experimental techniques and although the added mass coefficient has been determined for a wide range of shapes of bodies, no new physical meaning has been attached to it, and the basic definition of virtual mass has remained as: "The quotient of the force required to produce the accelerations throughout the fluid divided by the acceleration of the body." The usual derivation of

virtual mass is from kinetic energy which suffers from the inherent weakness that it assumes a constant velocity for the confined solid body, whereas all outward manifestations of mass should be associated with acceleration. Therefore, the velocity must be allowed to vary and the fact should be recognized that the added mass can yield momentum as well as energy.

Surprisingly enough, as early as 1888, Riecke[6] recognized the essential fact that for a sphere moving with constant velocity through an infinite inviscid fluid, the individual fluid particles which are pushed aside by the sphere in its forward motion do not return to their former positions. The paths of the individual particles are not closed curves but of the shape similar to that shown in Fig. 1, of this paper. Hence, besides pushing the particles aside temporarily in passing, the sphere also displaces the fluid particles permanently in the direction of its motion. However, the importance of this displacement and its relation to added mass was not recognized by Riecke. Recently, Darwin[7] has shown that this permanently displaced mass of the fluid enclosed between the initial and final positions of fluid particles,

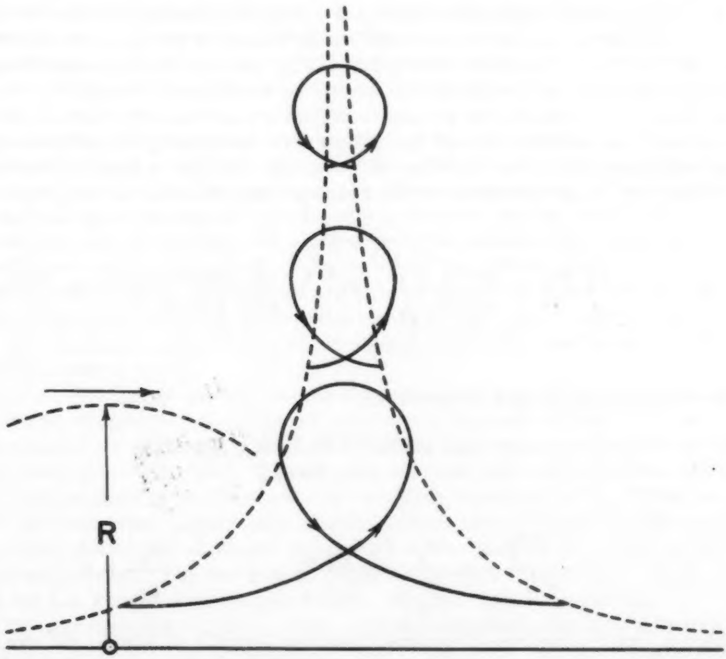


FIG. I

R is cylinder's radius. Trajectories are shown for three particles. Broken lines show the initial and final positions of particles before and after the passage of the cylinder.

is in fact the added mass itself. It can be shown, in general, that the motion of a body through an inviscid fluid media is always accompanied by a fluid-mass transport and that this mass is the added mass which unveils itself only if the body is accelerated.

The above explanation, although adding considerably to our understanding of the added mass, does not take into account the effect of the wake- or the cavity-induced mass. Furthermore, for an accelerating or decelerating body the induced mass varies with the instantaneous shape and volume of the wake or cavity as well as with their rates of change, i.e., the instantaneous magnitude of added mass values in transient conditions depends upon the time history of motion. Suggestions have been made to approximate transient motions by parts of a sine curve and thus to determine a frequency parameter. This procedure is obviously questionable. It is clear from the above discussion that the variation of the wake geometry with time causes marked changes in both C_d and C_m. Because both coefficients depend upon the state of development of the wake, recently McNown,[8] McNown and Keulegan[9] related them to each other quite successfully.

It should be noted, furthermore, that in an oscillatory gravity wave the elliptical orbits are open and there is a continuous mass transport in the direction of wave propagation near the surface, and in the opposite direction near the bottom. Therefore, the value of C_m computed from experimental data also involves the effect of the inertia of wave mass transport.

4. In order to obtain the pressure difference on the two sides of the volume of fluid which replaces the submerged structure, the authors could have used their Eq. (11), together with Eq. (4). If this is done for the case of a rectangular barge centered at the quarter point of a wave, one obtains

$$p_1 - p_2 = \frac{\rho V}{A_x}\frac{\partial u_x}{\partial t} = \frac{\rho V}{A_x}\frac{1}{L'}\int_{\frac{L}{4}-\frac{L'}{2}}^{\frac{L}{4}+\frac{L'}{2}} \frac{2\pi^2 H}{T^2}\frac{\cosh 2\pi(d+z)/L}{\sinh 2\pi d/L}\sin\frac{2\pi x}{L}\,dx \qquad (1)$$

or performing the simple integration,

$$p_1 - p_2 = wH\frac{\cosh 2\pi(d+z)/L}{\cosh 2\pi d/L}\cos\frac{2\pi x_1}{L} \qquad (2)$$

or

$$\frac{p_1 - p_2}{wH} = \frac{a_z}{a_o}\cos\frac{2\pi x_1}{L}, \qquad \frac{a_z}{a_o} = K$$

in which a_z and a_o represent respectively the lengths of horizontal axes of water particle orbits at a point z, and at a point on the still water level.

If Eq. (2) is compared with the authors' Eq. (16), it will be noticed that the second term in the parenthesis of the latter equation is missing in the former equation. The difference is basically due to the linearized averaging of the horizontal accelerations along the submerged structure. An exact analysis made by this writer of the difference in the pressures occurring under a crest and a trough using Struik's equations[10] has shown that the right side of the authors' Eq. (18), contains more negative terms in the form of a series

with rapidly decreasing absolute magnitudes. This analysis is not given here partly because it is irrelevant to this discussion and partly because it is rather lengthy.

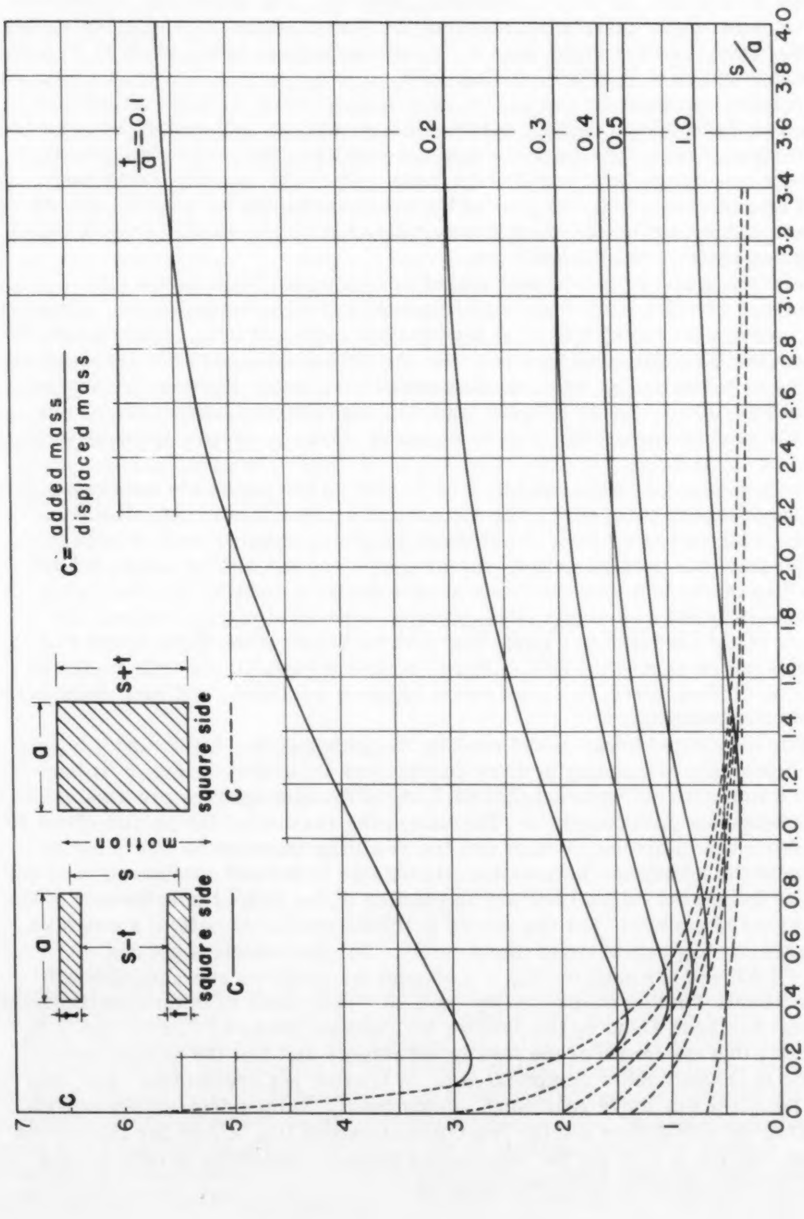
5. The authors have found that the reduction in displaced mass of the barge by introducing a slot is partially offset by, "the additional disturbance attributable to the odd configuration of the slotted barge . . .", and the values of C_m are greater than the ones for a full rectangular barge by 10 to 24 per cent for values of H/L from 0.02 to 0.06.

A study, made by this writer, to determine the virtual mass of multiple spheres, lens shaped bodies, and of circular, square, and rectangular plates of various sizes amply supports only the results of the above observation, but not the explanation given for its cause. Since the results of this study will be reported later, only part of the results obtained for parallel square plates and for sufficiently long (large ratio of length to width) parallel rectangular plates will be given here.

In Figs. 2 and 3, curves with dotted lines give the ratio (added mass/displaced mass) for full right square prism and for unslotted right rectangular prism, as a function of (s/a) for constant values of (t/a). Curves with full lines give the aforementioned ratio for the separated square and rectangular plates as a function of the same dimensionless ratios. Here, only the representative curves, rather than the separate experimental points, are shown for the sake of simplicity. The direction of vibratory motion is shown on the figures.

An examination of the figures reveals that as the plates are separated from each other but a very little distance, the added mass coefficient decreases slightly, i.e., the fluid between the plates behaves as if it were part of the solid full parallelepiped. As the plates are set farther apart, the added mass coefficient increases, but always remains smaller than the added mass coefficient of a single plate. Hence, when the distance between the plates is not too large in comparison with the width of the plate, there is a mutual interaction of the flow patterns around the two plates. As the plates become farther apart, this interaction becomes negligible and each plate behaves independently.

For the slotted barge model used by the authors $t/a = 1.265$, and $s/a = 4$. If the point corresponding to these parameters is located on Fig. 3, it becomes immediately apparent that each leg of the slot behaves practically as an independent parallelepiped. Therefore, the reason for the partial offset of the reduction in displaced mass and the resulting increase in C_m is not because of the additional disturbance attributable to the odd configuration of the slotted barge, but on the contrary is because of the lack of interference of the two sides of the slot. For the forces in the horizontal direction, a weighted average for C_m can be determined easily. For the unslotted portion $C_m = 1.33$, and for each leg $C_m = 1.95$, and the weighted average, although approximate for this shape of body, is 1.55. This value is in the neighborhood of what has been found by the authors for various values of T, z/d, and H/L. The fact that the length of the legs is not infinite and that the slotted barge model is actually more complicated is, of course, not overlooked. But, this will not alter the above reasoning. Using the C_m values determined experimentally by this writer for the two types of barges ($C_m = 1.65$ for the slotted barge, and $C_m = 1.35$ for the rectangular barge) a reduction of only 9.5 per cent in the horizontal force is computed.


C = added mass / displaced mass
t/a = 0.1
0.2
0.3
0.4
0.5
1.0
s/a
C
a
s−t
s
motion
s+t
square side
C ———
C - - - - -
0
1
2
3
4
5
6
7
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
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4.0

FIG. 2

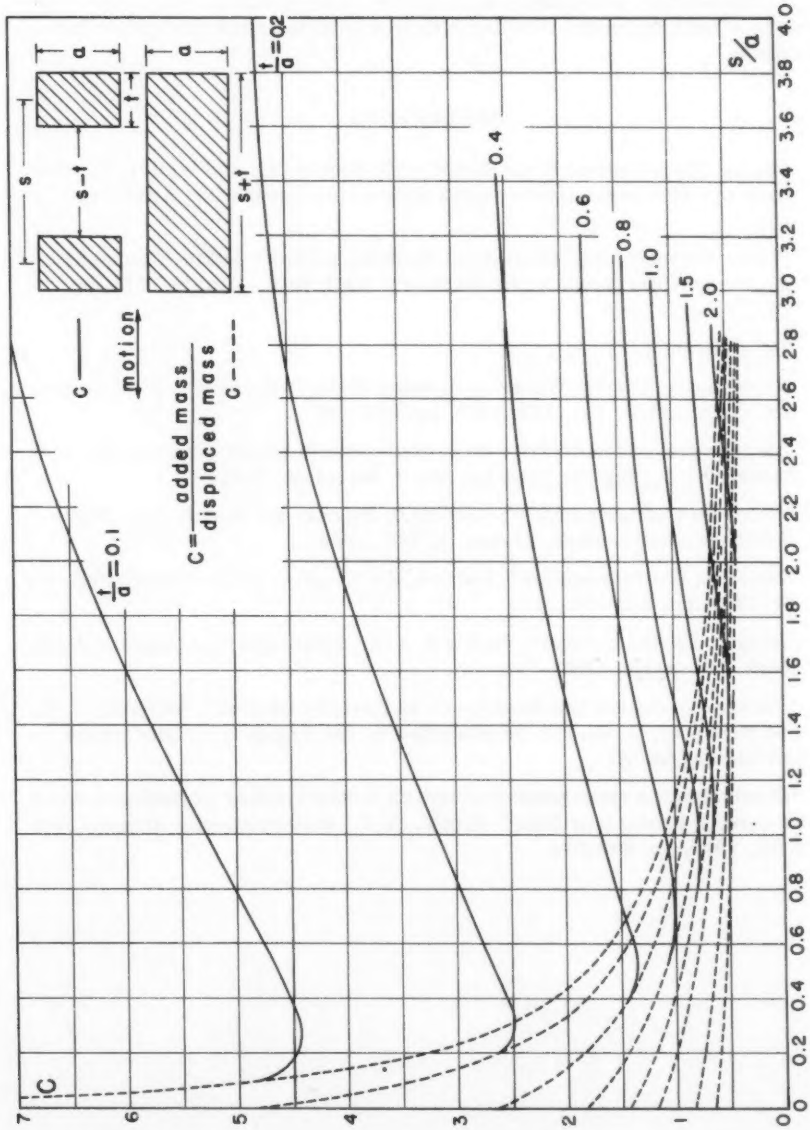
C
s/a
$\frac{t}{a}=0.1$
$\frac{t}{a}=0.2$
0.4
0.6
0.8
1.0
1.5
2.0
C ——
motion
$C=\frac{\text{added mass}}{\text{displaced mass}}$
C - - - -
s
s-t
a
t
s+t
0 1 2 3 4 5 6 7
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0

FIG. 3

Corrections.—In Eq. (1), a Pi term is missing due to an obvious oversight. The equations for the horizontal and vertical components of the orbital velocity (Eqs. (4) and (5)), and the equations for horizontal and vertical accelerations (Eqs. (6) and (7)), and hence, the tabulated values in Fig. 5, are not mathematically consistent as far as the rules of differentiation are concerned and as long as the meaning attached to theta is not changed in the meantime.

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PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1859 is identified as 1859 (HY 7) which indicates that the paper is contained in the seventh issue of the Journal of the Hydraulics Division during 1958.

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MARCH: 1560(ST2), 1561(ST2), 1562(ST2), 1563(ST2), 1564(ST2), 1565(ST2), 1566(ST2), 1567(ST2), 1568(WW2), 1569(WW2), 1570(WW2), 1571(WW2), 1572(WW2), 1573(WW2), 1574(PL1), 1575(PL1), 1576(ST2)[c], 1577(PL1), 1578(PL1)[c], 1579(WW2)[c].

APRIL: 1580(EM2), 1581(EM2), 1582(HY2), 1583(HY2), 1584(HY2), 1585(HY2), 1586(HY2), 1587(HY2), 1588(HY2), 1589(IR2), 1590(IR2), 1591(IR2), 1592(SA2), 1593(SU1), 1594(SU1), 1595(SU1), 1596(EM2), 1597(PO2), 1598(PO2), 1599(PO2), 1600(PO2), 1601(PO2), 1602(PO2), 1603(HY2), 1604(EM2), 1605(SU1)[c], 1606(SA2), 1607(SA2), 1608(SA2), 1609(SA2), 1610(SA2), 1611(SA2), 1612(SA2), 1613(SA2), 1614(SA2)[c], 1615(IR2)[c], 1616(HY2)[c], 1617(SU1), 1618(PO2)[c], 1619(EM2)[c], 1620(CP1).

MAY: 1621(HW2), 1622(HW2), 1623(HW2), 1624(HW2), 1625(HW2), 1626(HW2), 1627(HW2), 1628(HW2), 1629(ST3), 1630(ST3), 1631(ST3), 1632(ST3), 1633(ST3), 1634(ST3), 1635(ST3), 1636(ST3), 1637(ST3), 1638(ST3), 1639(WW3), 1640(WW3), 1641(WW3), 1642(WW3), 1643(WW3), 1644(WW3), 1645(SM2), 1646(SM2), 1647(SM2), 1648(SM2), 1649(SM2), 1650(SM2), 1651(HW2), 1652(HW2)[c], 1653(WW3)[c], 1654(SM2), 1655(SM2), 1656(ST3)[c], 1657(SM2)[c].

JUNE: 1658(AT1), 1659(AT1), 1660(HY3), 1661(HY3), 1662(HY3), 1663(HY3), 1664(HY3), 1665(SA3), 1666(PL2), 1667(PL2), 1668(PL2), 1669(AT1), 1670(PO3), 1671(PO3), 1672(PO3), 1673(PL2), 1674(PL2), 1675(PO3), 1676(PO3), 1677(SA3), 1678(SA3), 1679(SA3), 1680(SA3), 1681(SA3), 1682(SA3), 1683(PO3), 1684(HY3), 1685(SA3), 1686(SA3), 1687(PO3), 1688(SA3)[c], 1689(PO3)[c], 1690(HY3)[c], 1691(PL2)[c].

JULY: 1692(EM3), 1693(EM3), 1694(ST4), 1695(ST4), 1696(ST4), 1697(SU2), 1698(SU2), 1699(SU2), 1700(SU2), 1701(SA4), 1702(SA4), 1703(SA4), 1704(SA4), 1705(SA4), 1706(EM3), 1707(ST4), 1708(ST4), 1709(ST4), 1710(ST4), 1711(ST4), 1712(ST4), 1713(SU2), 1714(SA4), 1715(SA4), 1716(SU2), 1717(SA4), 1718(EM3), 1719(EM3), 1720(SU2), 1721(ST4)[c], 1722(ST4), 1723(ST4), 1724(EM3)[c].

AUGUST: 1725(HY4), 1726(HY4), 1727(SM3), 1728(SM3), 1729(SM3), 1730(SM3), 1731(SM3), 1732(SM3), 1733(PO4), 1734(PO4), 1735(PO4), 1736(PO4), 1737(PO4), 1738(PO4), 1739(PO4), 1740(PO4), 1741(PO4), 1742(PO4), 1743(PO4), 1744(PO4), 1745(PO4), 1746(PO4), 1747(PO4), 1748(PO4), 1749(PO4).

SEPTEMBER: 1750(IR3), 1751(IR3), 1752(IR3), 1753(IR3), 1754(IR3), 1755(ST5), 1756(ST5), 1757(ST5), 1758(ST5), 1759(ST5), 1760(ST5), 1761(ST5), 1762(ST5), 1763(ST5), 1764(ST5), 1765(WW4), 1766(WW4), 1767(WW4), 1768(WW4), 1769(WW4), 1770(WW4), 1771(WW4), 1772(WW4), 1773(WW4), 1774(IR3), 1775(IR3), 1776(SA5), 1777(SA5), 1778(SA5), 1779(SA5), 1780(SA5), 1781(WW4), 1782(SA5), 1783(SA5), 1784(IR3)[c], 1785(WW4)[c], 1786(SA5)[c], 1787(ST5)[c], 1788(IR3), 1789(WW4).

OCTOBER: 1790(EM4), 1791(EM4), 1792(EM4), 1793(EM4), 1794(EM4), 1795(HW3), 1796(HW3), 1797(HW3), 1798(HW3), 1799(HW3), 1800(HW3), 1801(HW3), 1802(HW3), 1803(HW3), 1804(HW3), 1805(HW3), 1806(HY5), 1807(HY5), 1808(HY5), 1809(HY5), 1810(HY5), 1811(HY5), 1812(SM4), 1813(SM4), 1814(ST6), 1815(ST6), 1816(ST6), 1817(ST6), 1818(ST6), 1819(ST6), 1820(ST6), 1821(ST6), 1822(EM4), 1823(PO5), 1824(SM4), 1825(SM4), 1826(SM4), 1827(ST6)[c], 1828(SM4)[c], 1829(HW3)[c], 1830(PO5)[c], 1831(EM4)[c], 1832(HY5)[c].

NOVEMBER: 1833(HY6), 1834(HY6), 1835(SA6), 1836(ST7), 1837(ST7), 1838(ST7), 1839(ST7), 1840(ST7), 1841(ST7), 1842(SU3), 1843(SU3), 1844(SU3), 1845(SU3), 1846(SU3), 1847(SA6), 1848(SA6), 1849(SA6), 1850(SA6), 1851(SA6), 1852(SA6), 1853(SA6), 1854(ST7), 1855(SA6)[c], 1856(HY6)[c], 1857(ST7)[c], 1858(SU3)[c].

DECEMBER: 1859(HY7), 1860(IR4), 1861(IR4), 1862(IR4), 1863(SM5), 1864(SM5), 1865(ST8), 1866(ST8), 1867(ST8), 1868(PP1), 1869(PP1), 1870(PP1), 1871(PP1), 1872(PP1), 1873(WW5), 1874(WW5), 1875(WW5), 1876(WW5), 1877(CP2), 1878(ST8), 1879(ST8), 1880(HY7)[c], 1881(SM5)[c], 1882(ST8)[c], 1883(PP1)[c], 1884(WW5)[c], 1885(CP2)[c], 1886(PO6), 1887(PO6), 1888(PO6), 1889(PO6), 1890(HY7), 1891(PP1).

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MARCH: 1960(HY3), 1961(ST3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968(SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977(WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986(IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

c. Discussion of several papers, grouped by divisions.

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